

Data Shapley:

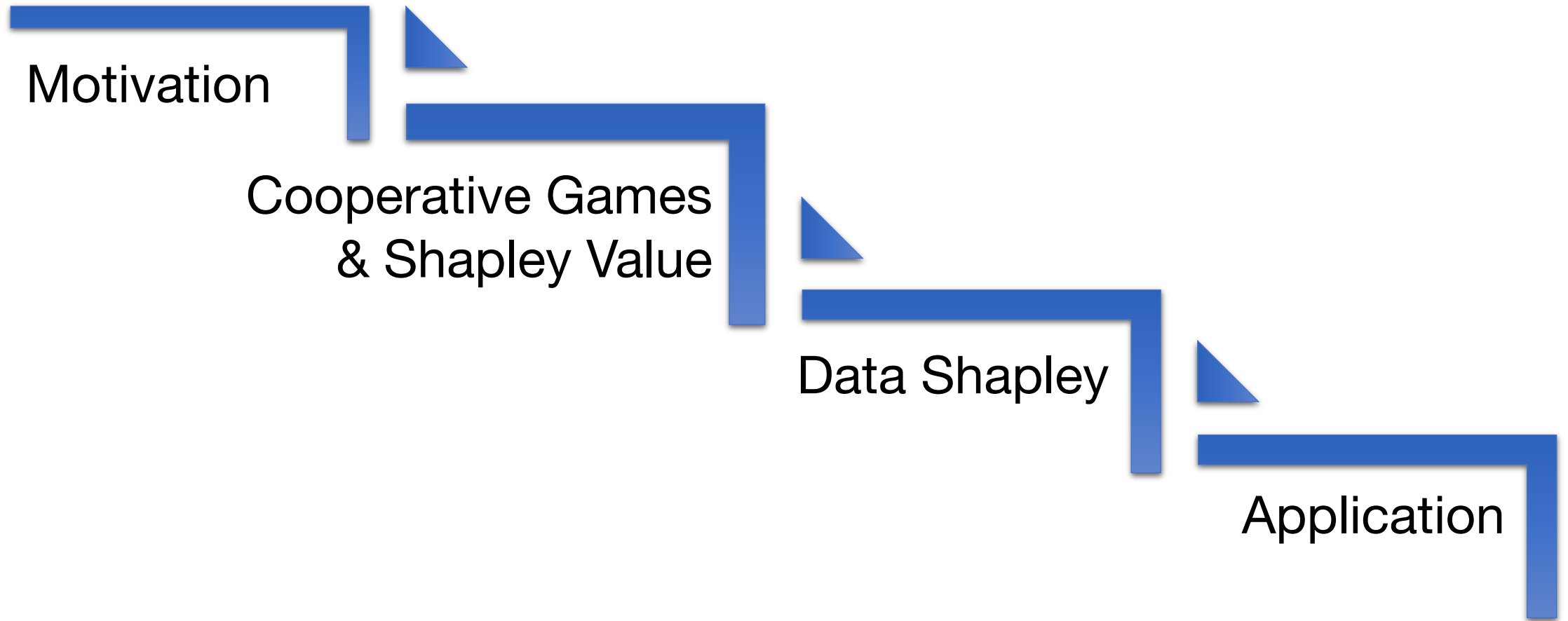
Equitable Valuation of Data for Machine Learning

Amirata Ghorbani, Michael P. Kim, James Zou

2019



Overview



Collaborative Machine Learning

- **Data** is the fuel powering machine learning.
- Where does data come from?

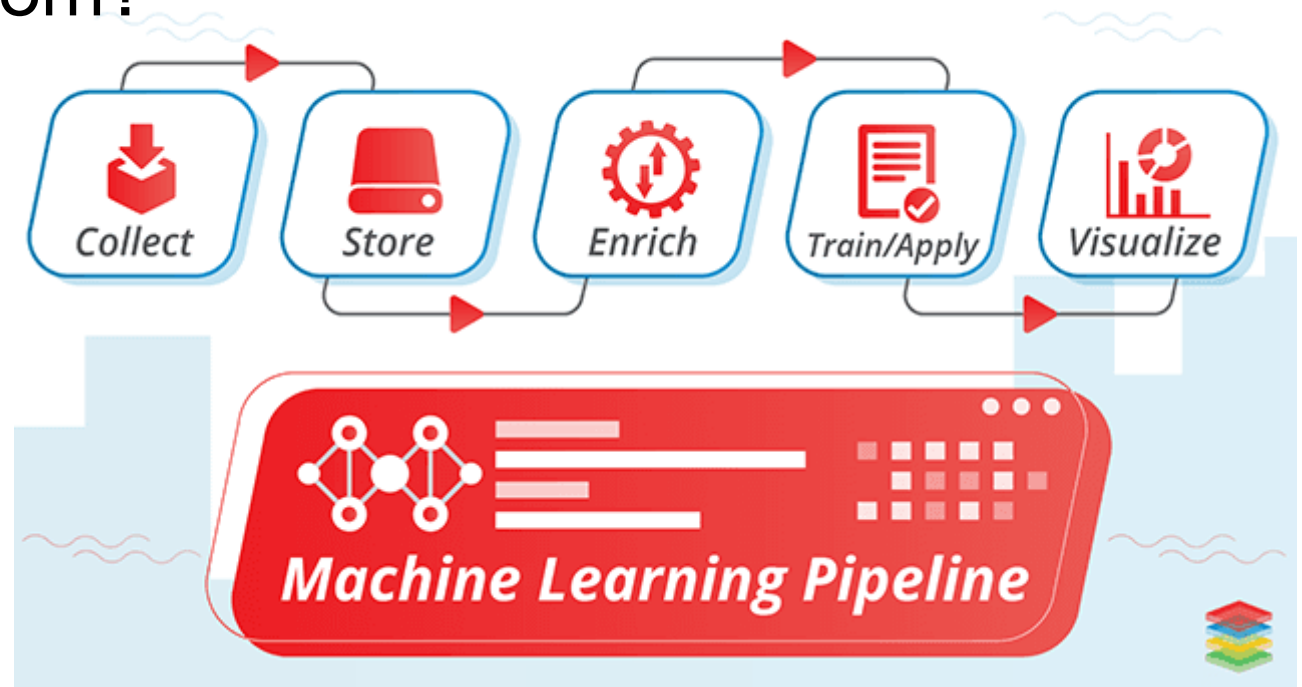


Figure: Machine Learning Pipeline (Gill, 2022).



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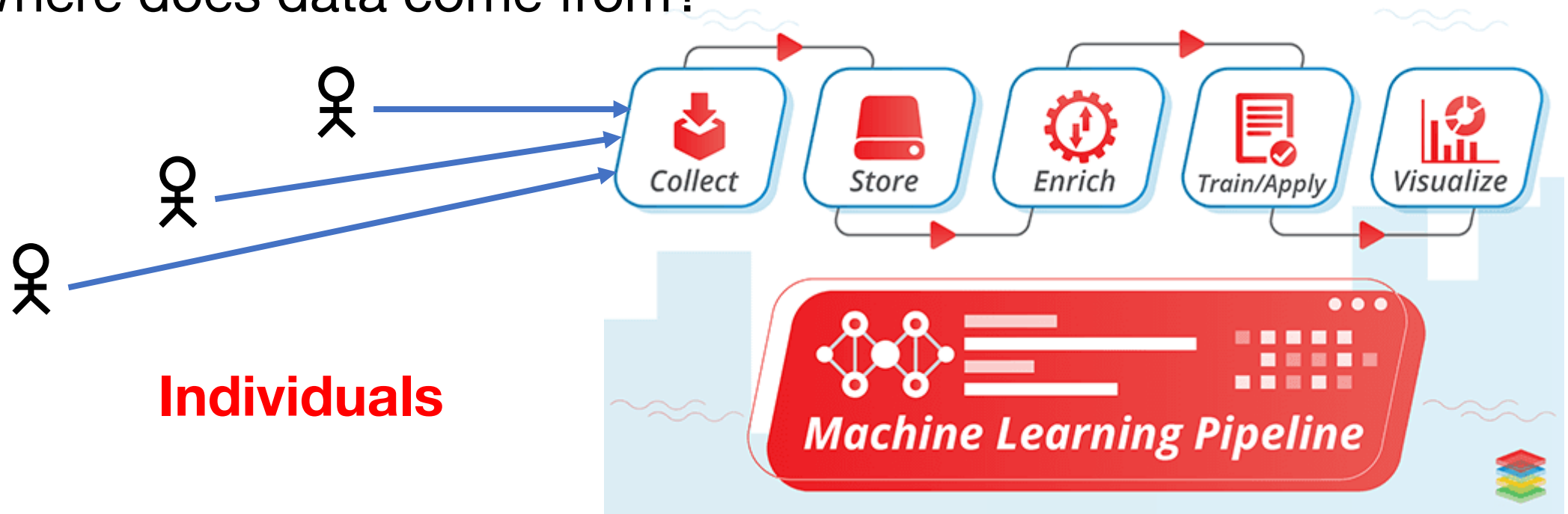


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General Data Protection Regulation

- **Data** are properties. Properties are not **free** for use.

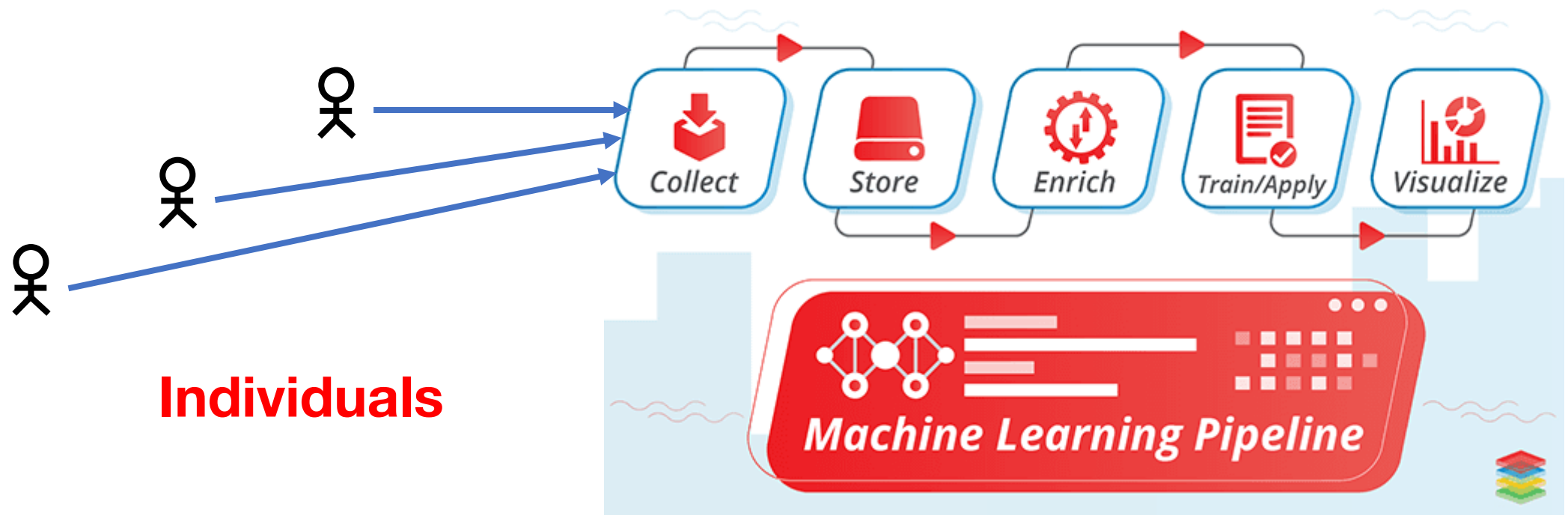


Figure: Machine Learning Pipeline (Gill, 2022).



Data Valuation

- Need to assign a value to each individual's data so that everyone is fairly compensated.

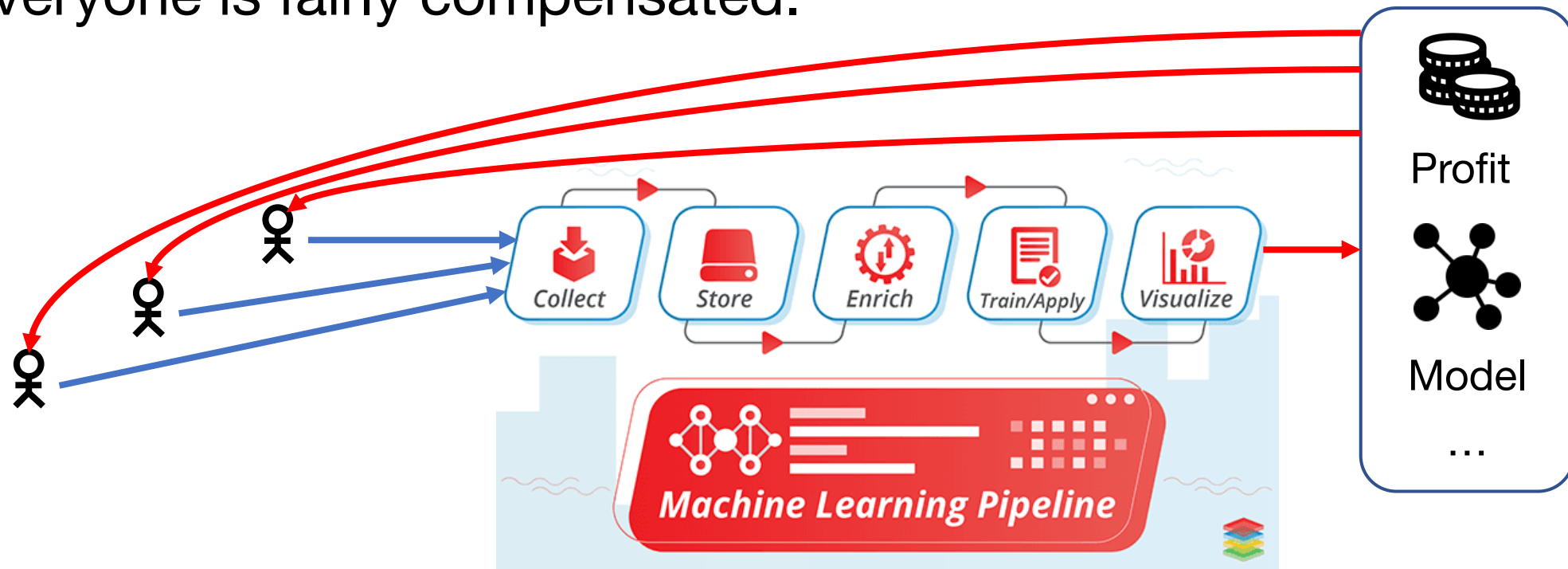
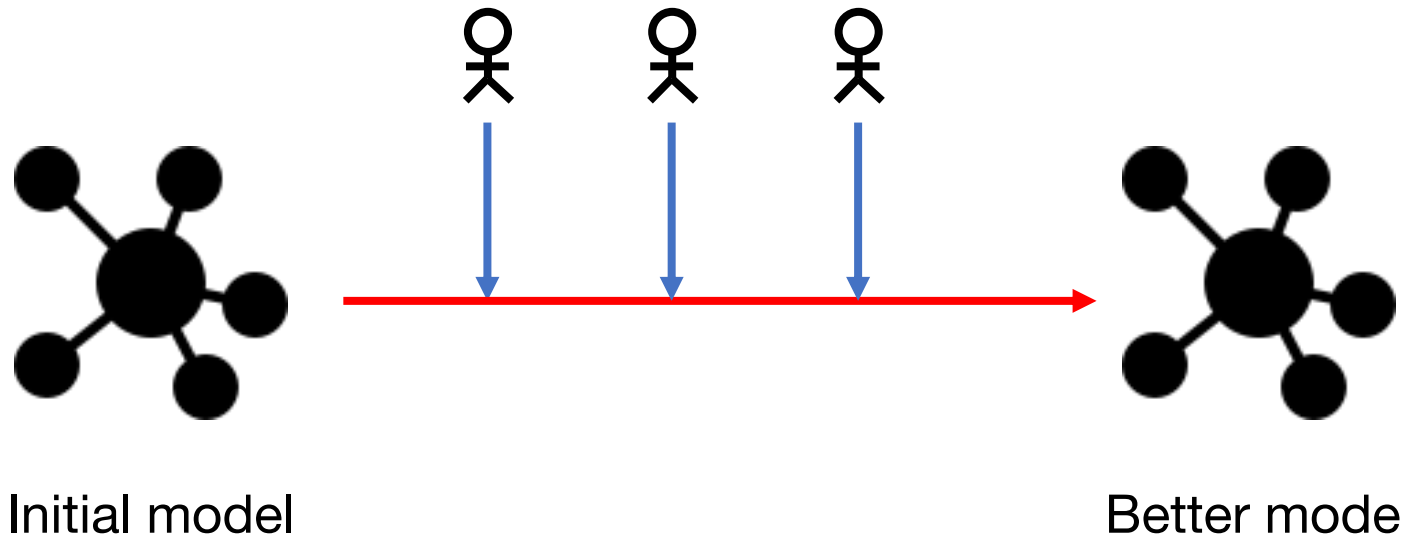


Figure: Machine Learning Pipeline (Gill, 2022).



A cooperative game!

- Through cooperation, we obtain a **better** model than without cooperation.



Evaluation metrics

- Accuracy
- MSE
- F1 score
- Information gain
- ...



Game Theory

Traditional

- Players are **rational** and **selfish**.
- “Prisoner's Dilemma”: Both prisoners will eventually choose to **defect** because whatever the other prisoner choose, to defect gives the better outcome.



Figure: Prisoner's Dilemma (Forsythe, 2012).

Game Theory

Traditional

- Players are **rational** and **selfish**.
- “Prisoner's Dilemma”: Both prisoners will eventually choose to **defect** because whatever the other prisoner choose, to defect gives the better outcome.

But this is not the best outcome!



Figure: Prisoner's Dilemma (Forsythe, 2012).

Game Theory

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- Players are **rational** and **selfish**.
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Cooperative

- Players have **common interests**, **information exchange** and **compulsory contract**.
- Both prisoners should **not** defect to gain mutual benefits.

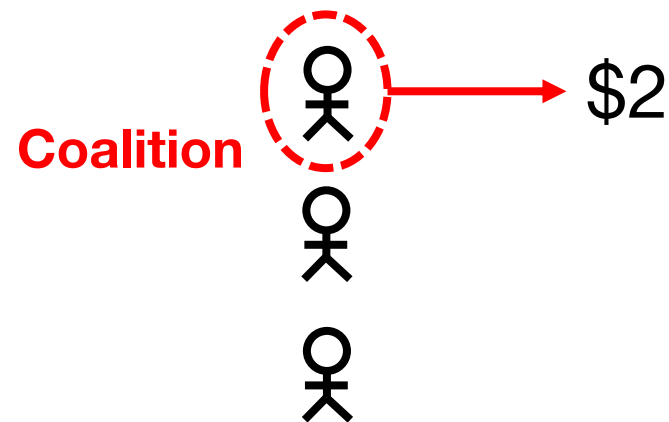


Cooperative Games

- A **game** is uniquely defined by a set function

$$V: 2^N \rightarrow \mathbb{R} \quad \text{aka Value Function}$$

where N represents the set of players in the game.

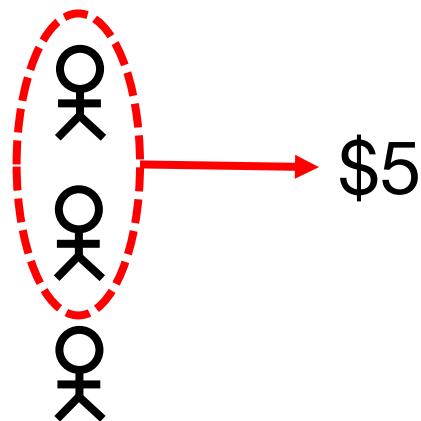


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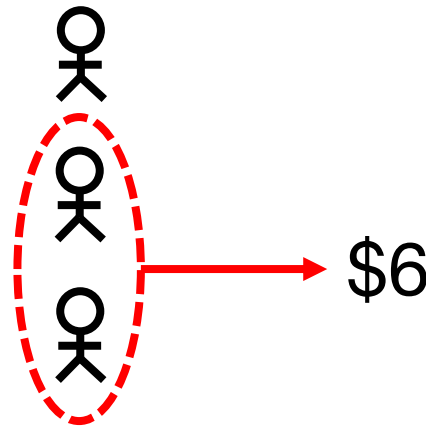


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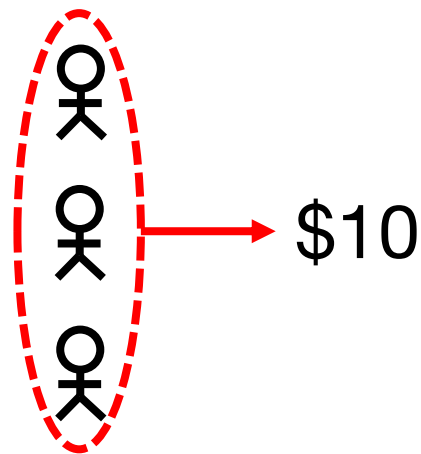


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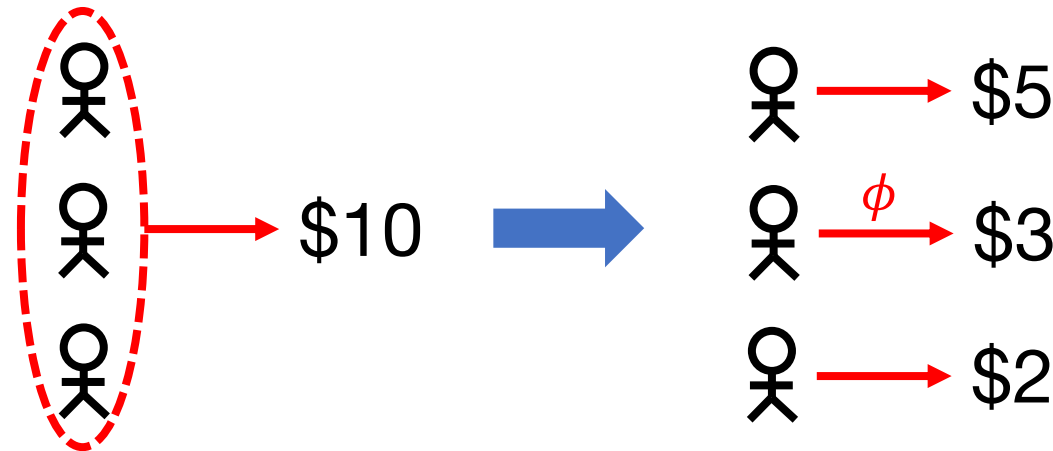


Contribution Function

- To measure the contribution of each player, we define

$$\phi_V: N \rightarrow \mathbb{R}$$

where N represents the set of players in the game.



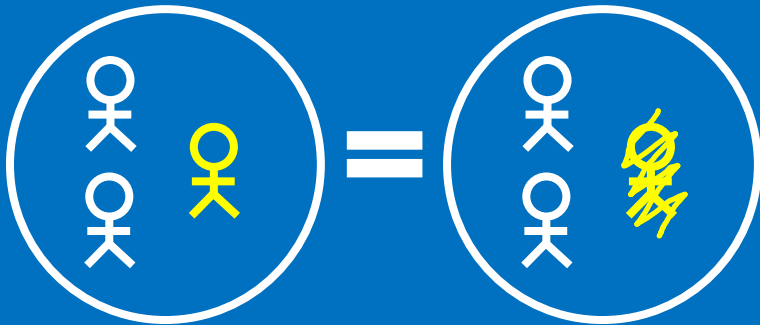
Fair Measure of Contribution



Analogy: Measure the value of a new colleague in the workplace.

Null Player

When player i joins any existing work group, he does not add value to that group.



$$\forall S \subseteq N \setminus \{i\} [V(S) = V(S + \{i\})] \\ \Rightarrow \phi(i) = 0$$



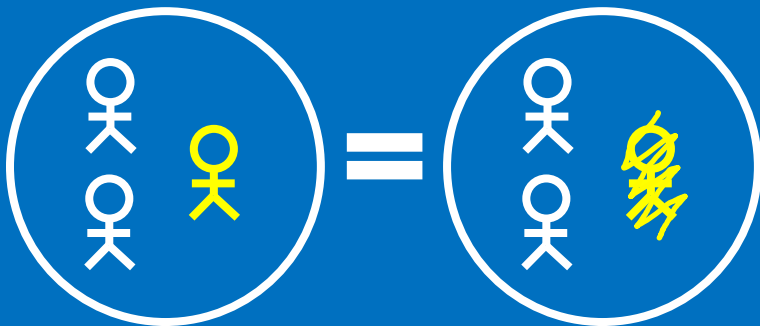
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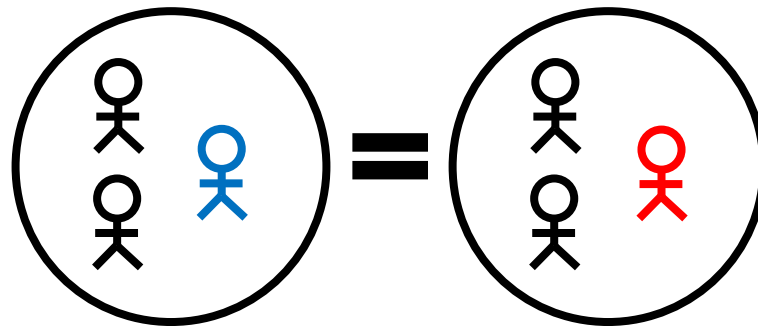
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Symmetry

When player i and j join any existing work group, they add the same value to that group.



$$\forall S \subseteq N \setminus \{i, j\} [V(S + \{i\}) \\ = V(S + \{j\})] \Rightarrow \phi(i) = \phi(j)$$



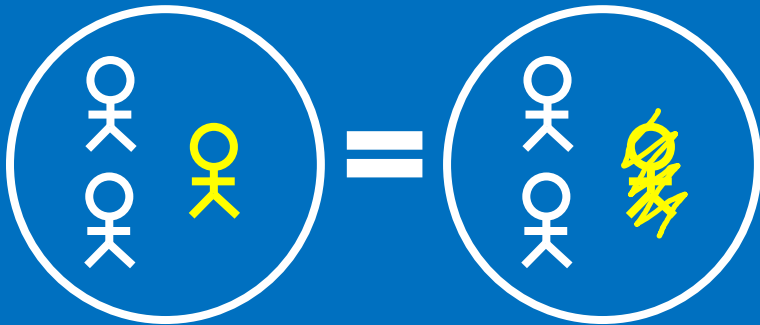
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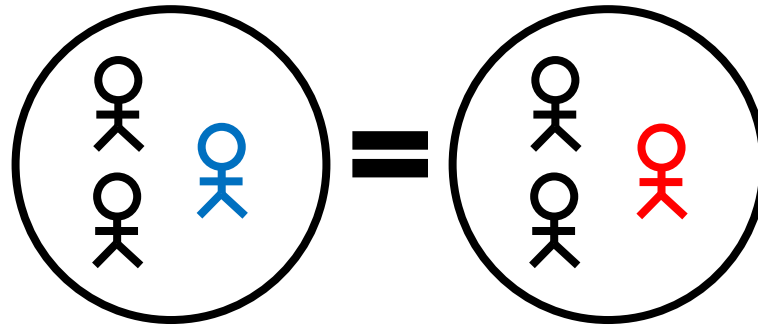
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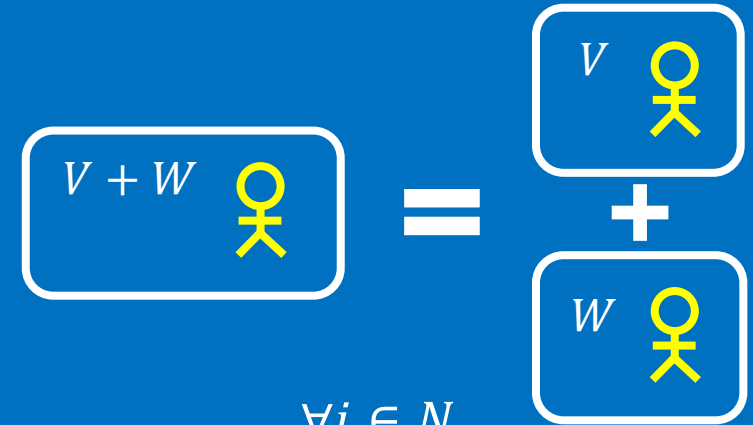
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Linearity

We have two scores V and W for each work group. We take the combined score as $V + W$.



$$\forall i \in N \\ [\phi_V(i) + \phi_W(i) = \phi_{V+W}(i)]$$



Shapley Value

- Shapley found such a value:

$$\phi(i) = \frac{1}{|N|} \sum_{S \subseteq N \setminus \{i\}} \frac{V(S + \{i\}) - V(S)}{\binom{n-1}{|S|}}$$

- Besides Null Player, Symmetry and Linearity, the Shapley value is special such that it is the only one that satisfies **Efficiency**:

$$\sum_{i \in N} \phi(i) = V(N)$$



Figure: Lloyd S. Shapley (Moreno et al., 2018).



Shapley Value

**Marginal
contribution**

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Data Shapley

Performance metrics/Information gain



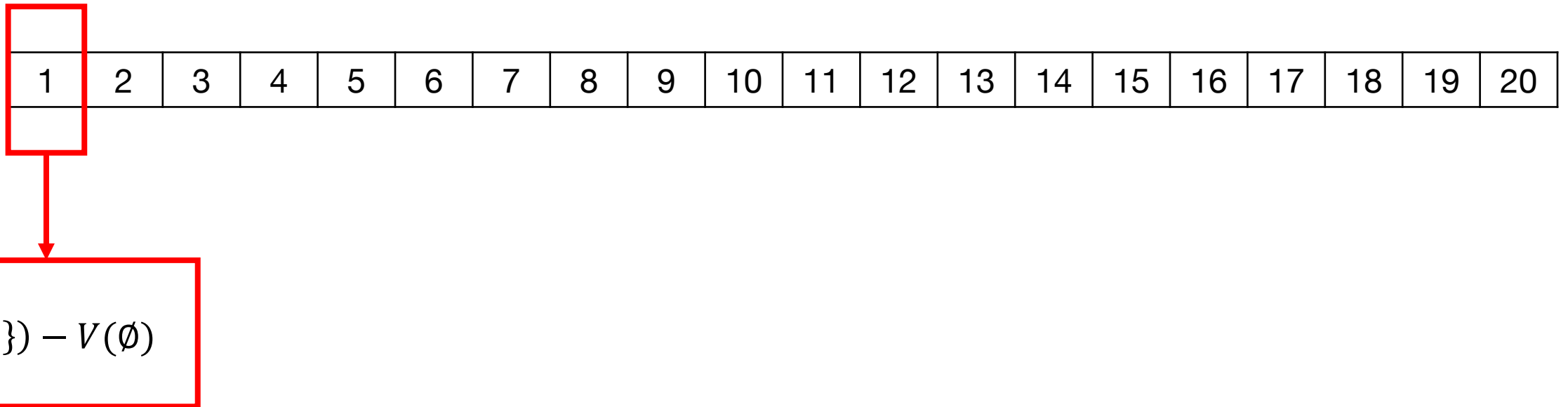
$$\phi(i) = c \sum_{S \subseteq N \setminus \{i\}} \frac{V(S + \{i\}) - V(S)}{\binom{n-1}{|S|}}$$

- S is every subset of N , leading to **very high computational cost** (in machine learning, we usually have millions of data!).



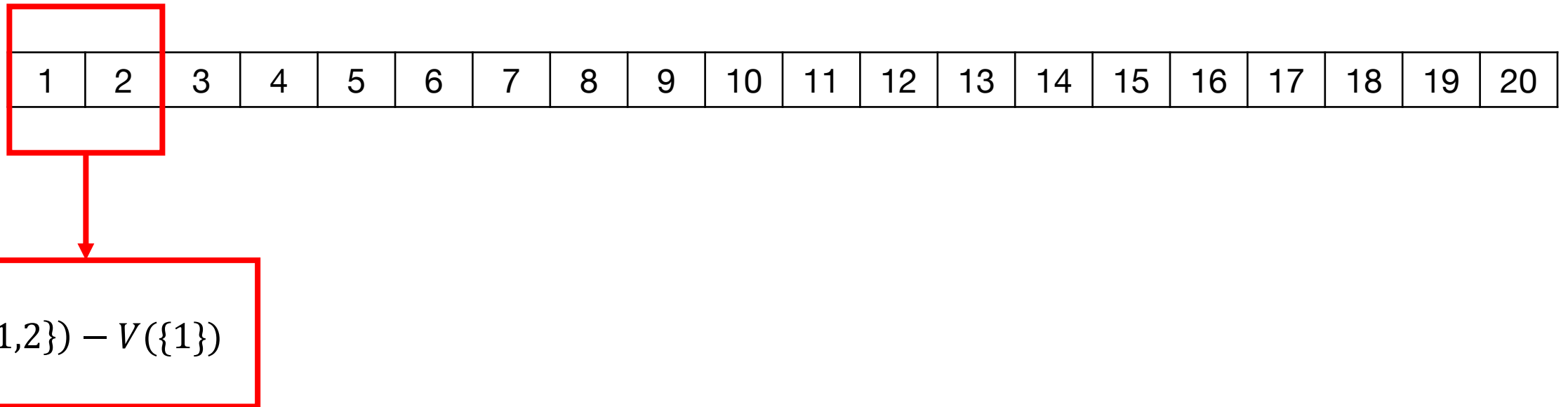
Truncated Monte Carlo (TMC) Shapley

- General idea I: Take a random permutation of data and calculate the marginal contribution in a **rolling** basis.



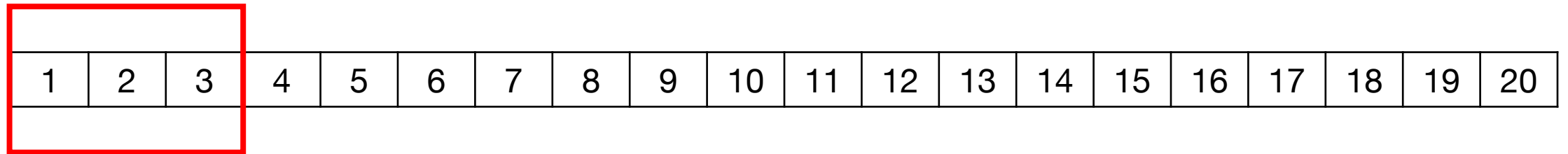
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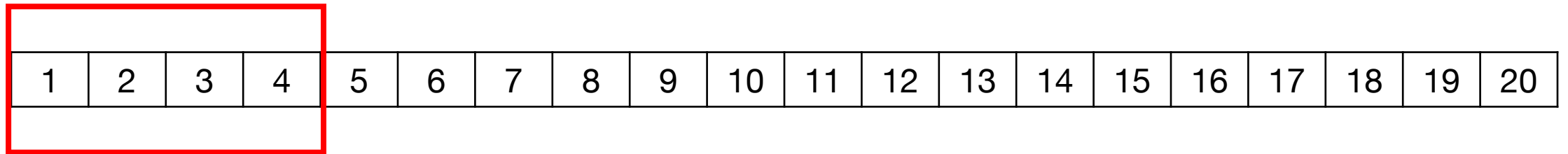


$$V(\{1,2,3\}) - V(\{1,2\})$$



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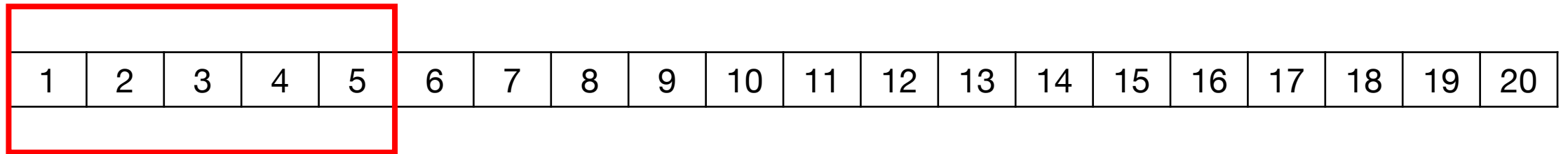


$$V(\{1,2,3,4\}) - V(\{1,2,3\})$$



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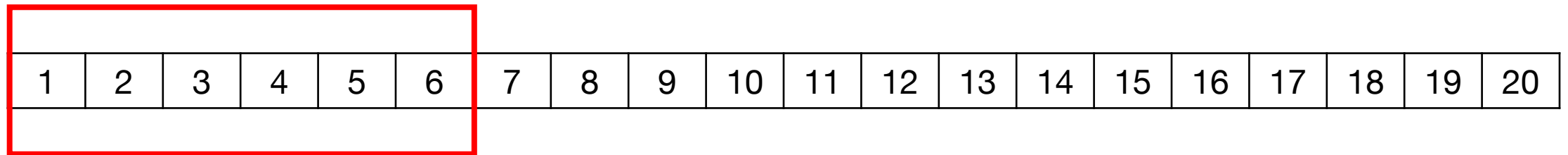


$$V(\{1,2,3,4,5\}) - V(\{1,2,3,4\})$$



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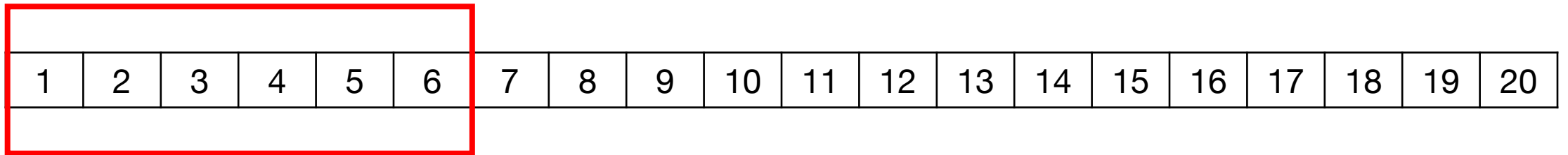


$$V(\{1,2,3,4,5,6\}) - V(\{1,2,3,4,5\})$$



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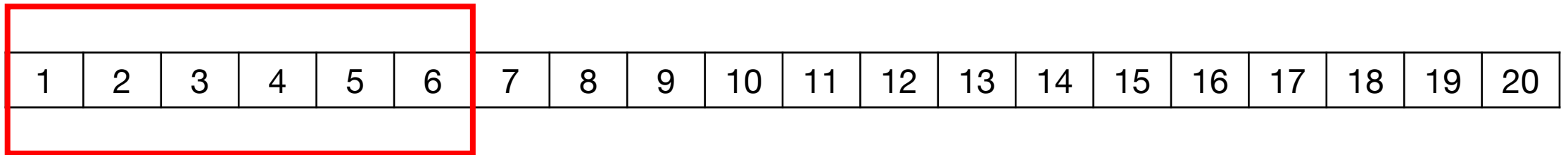


$$V(\{1,2,3,4,5,6\}) - V(\{1,2,3,4,5\}) < \epsilon$$



Truncated Monte Carlo (TMC) Shapley

- General idea II: When the marginal contribution becomes very small, mark all the remaining contribution as 0.

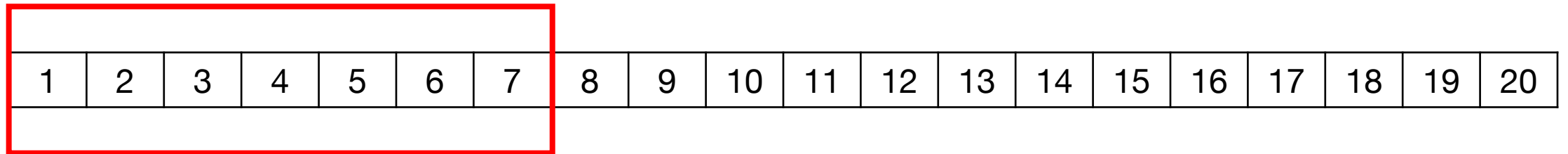


$$V(\{1,2,3,4,5,6\}) - V(\{1,2,3,4,5\}) = 0$$



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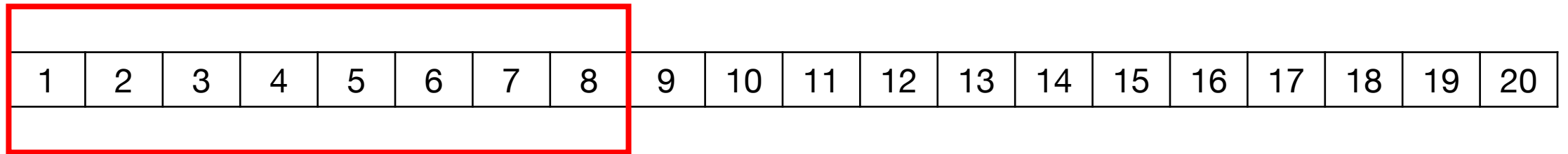


$$V(\{1,2,3,4,5,6,7\}) - V(\{1,2,3,4,5,6\}) = 0$$



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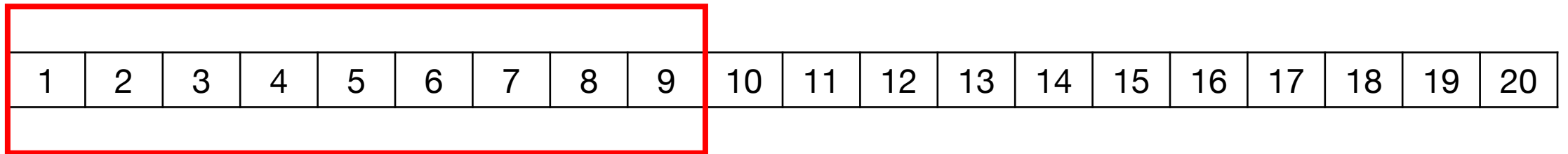


$$V(\{1,2,3,4,5,6,7,8\}) - V(\{1,2,3,4,5,6,7\}) = 0$$



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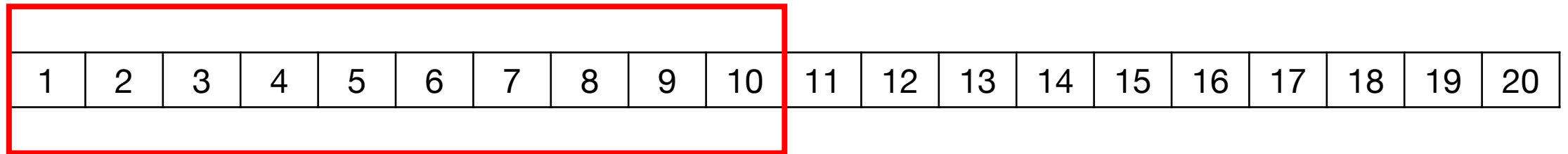


$$V(\{1,2,3,4,5,6,7,8,9\}) - V(\{1,2,3,4,5,6,7,8\}) = 0$$



Truncated Monte Carlo (TMC) Shapley

- General idea II: When the marginal contribution becomes very small, mark all the remaining contribution as 0.



$$V(\{1,2,3,4,5,6,7,8,9,10\}) - V(\{1,2,3,4,5,6,7,8,9\}) = 0$$



Application: Low Quality Data

Mislabelled data has
low (even -ve) Data Shapley value!

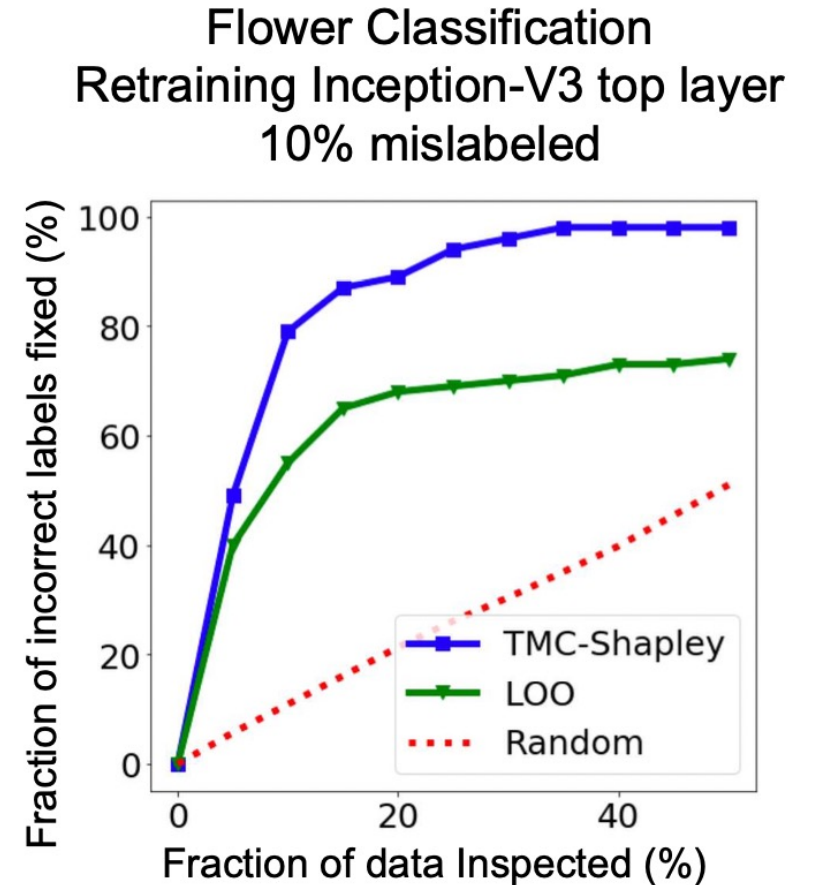


Figure: Identifying mislabelled data and correcting them (Ghorbani & Zou, 2018).



Application: Differentiate Data Sources

- “All data sources are not created equal.”s

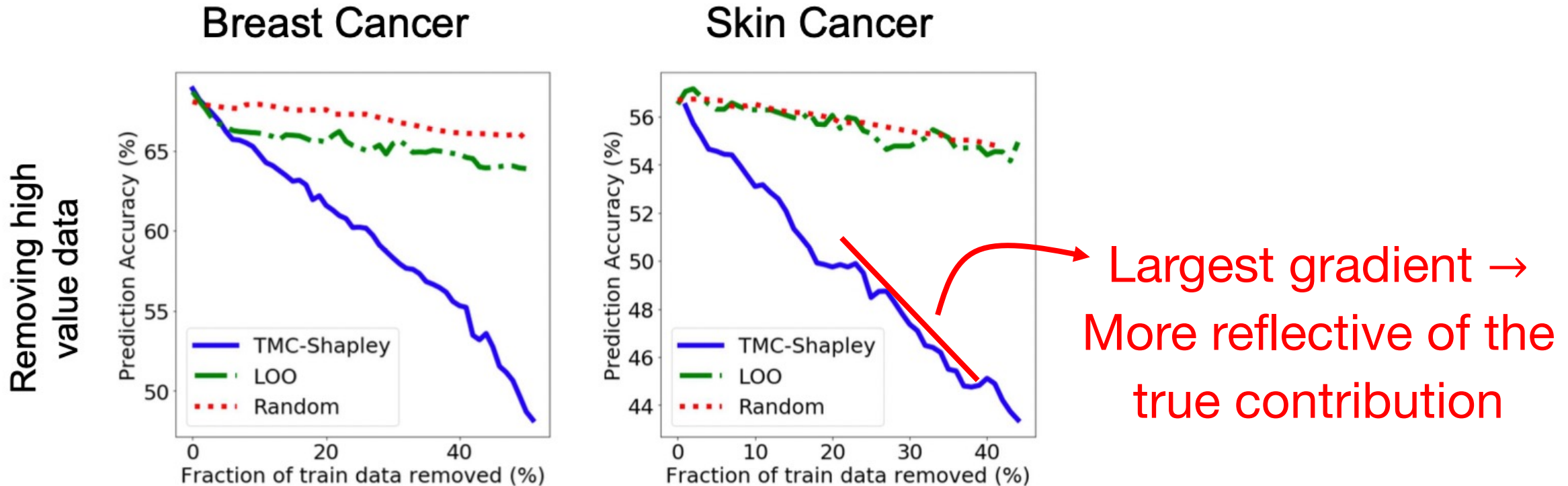


Figure: Change of prediction accuracy as high value data are removed gradually (Ghorbani & Zou, 2018).



Application: Adapt to New Data

1. Use performance metrics on **target data** as value function.
2. Remove –ve value data.
3. Use value of data as **weight** when training them.

Source to Target	Prediction Task	Trained Model	Original Performance (%)	Adapted Performance (%)
Google to HAM1000	Skin Lesion Classification	Retraining Inception-V3 top layer	29.6	37.8
CSU to PP	Disease Coding	Retraining DeepTag top layer	87.5	90.1
LFW+ to PPB	Gender Detection	Retraining Inception-V3 top layer	84.1	91.5
MNIST to UPS	Digit Recognition	Multinomial Logistic Regression	30.8	39.1
Email to SMS	Spam Detection	Naive Bayes	68.4	86.4

Figure: Original performance vs Data Shapley Adapted Performance on different prediction tasks (Ghorbani & Zou, 2018).



Related Works & Discussion

- **Cook's Distance** in Linear Regression
- **Leverage** and **Influence**

These quantities does not satisfy **Null Player**, **Symmetry** and **Linearity!**



References

Forsythe, G. (2012, December 4). *Prisoner's Dilemma*. Flickr.
<https://www.flickr.com/photos/gforsythe/8245423564>

Ghorbani, A., & Zou, J. (2019, May). Data Shapley: Equitable Valuation of Data for Machine Learning. In *International Conference on Machine Learning* (pp. 2242-2251). PMLR.

Ghorbani, A., Kim, M., & Zou, J. (2020, November). A distributional framework for data valuation. In *International Conference on Machine Learning* (pp. 3535-3544). PMLR.

Gill, N. S. (2022, August 19). *Machine Learning Pipeline Deployment and Architecture*. Xenonstack.
<https://www.xenonstack.com/blog/machine-learning-pipeline>

Jia, R., Sun, X., Xu, J., Zhang, C., Li, B., & Song, D. (2019). An empirical and comparative analysis of data valuation with scalable algorithms.

Koh, P. W., & Liang, P. (2017, July). Understanding black-box predictions via influence functions. In *International conference on machine learning* (pp. 1885-1894). PMLR.

Kwon, Y., & Zou, J. (2021). Beta Shapley: a unified and noise-reduced data valuation framework for machine learning. arXiv preprint arXiv:2110.14049.

Moreno, V., Ramírez M. E., Oliva C. D. L., & Moreno E. (2018, May 21). *Biografía de Lloyd S. Shapley*. Busca Biografías. <https://www.buscabiografias.com/biografia/verDetalle/9903/Lloyd%20S.%20Shapley>



Appendix: Leave-one-out (LOO) Value

$$LOO(i) = V(N) - V(N \setminus \{i\})$$

This is actually the marginal contribution to the grand coalition without i !

- Leave-one-out value is much **easier to compute** than the Shapley value, and it is **robust to clone**.



Appendix: Limitation of Data Shapley

- Still expensive in **time**!
- Data Shapley gives each cardinality a **uniform weight** $(\frac{1}{|N|})$. This is actually **suboptimal**!
- The 3 axioms used are not universally applicable.
- The Efficiency axiom is **not** important in ML setting 😊!



Appendix: Use \mathcal{C} instead of $\frac{1}{|N|}$

$$\phi(i) = \mathcal{C} \sum_{S \subseteq N \setminus \{i\}} \frac{V(S + \{i\}) - V(S)}{\binom{n-1}{|S|}}$$

- In data valuation, the **Efficiency** axiom is not that useful.
- \mathcal{C} can be any arbitrary constant representing the scale since it does not affect the relative weight between data points.



Appendix: Variants of Data Shapley

$$\phi(i) = \frac{1}{|N|} \sum_{S \subseteq N \setminus \{i\}} \frac{\text{marginal contribution of } i}{\binom{n-1}{|S|}}$$

- **Banzhaf index:** $\frac{1}{2^{|N|-1}} \sum_{S \subseteq N \setminus \{i\}} \text{marginal contribution of } i$
- **Beta Shapley:** $\frac{1}{|N|} \sum_{S \subseteq N \setminus \{i\}} w \cdot \frac{\text{marginal contribution of } i}{\binom{n-1}{|S|}}$, where $w \sim \text{Beta}(\alpha, \beta)$.
- **D-Shapley:** $\mathbb{E}_{D^{|N|}}(\phi(i))$