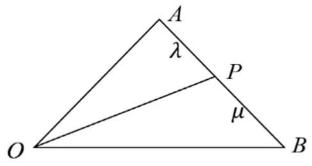
H2 Mathematics Contents

Vectors	• $\hat{\mathbf{u}} = \frac{\mathbf{u}}{\lvert\mathbf{u}\rvert}$ • A, B, C are collinear. $\Leftrightarrow$ $\overrightarrow{AB} = \lambda\overrightarrow{BC}$ for some real number λ. • Ratio Theorem: $\overrightarrow{OP} = \frac{\lambda\overrightarrow{OB}+\mu\overrightarrow{OA}}{\lambda+\mu}$  • $\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ • If $\mathbf{a}\perp\mathbf{b}$, $\mathbf{a}\cdot\mathbf{b} = 0$ • $\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = -(\mathbf{b}\times\mathbf{a})$ • $\mathbf{a}\times\mathbf{b} = (\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\mathbf{sin}\theta)\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the unit vector perpendicular to the plane containing **a** and **b**. • If $\mathbf{a}\parallel\mathbf{b}$, $\mathbf{a}\times\mathbf{b} = 0$ • Length of projection of **a** onto **b**: $\lvert\mathbf{a}\cdot\hat{\mathbf{b}}\rvert$ • Projection vector of **a** onto **b**: $(\mathbf{a}\cdot\hat{\mathbf{b}})\hat{\mathbf{b}}$ • Length of vector component of **a** perpendicular to **b**: $\lvert\mathbf{a}\times\hat{\mathbf{b}}\rvert$ • Vector component of **a** perpendicular to **b**: $\mathbf{a} - (\mathbf{a}\cdot\hat{\mathbf{b}})\hat{\mathbf{b}}$ • Equation of a line: $\begin{cases} \mathbf{r} = \mathbf{a} + \lambda\mathbf{m} \\ \frac{x-a_1}{m_1} = \frac{y-a_2}{m_2} = \frac{z-a_3}{m_3} \end{cases}$ • Equation of a plane: $\begin{cases} \mathbf{r} = \mathbf{a} + \lambda\mathbf{m_1} + \mu\mathbf{m_2} \\ \mathbf{r}\cdot\mathbf{n} = D \\ ax + by + cz = D \end{cases}$ • Angle between lines and planes: using $\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ • Distance from a point to a line or plane: using projection or perpendicular component. • Reflection of a point: using the foot of perpendicular from the point to the line or plane.
Curve Sketching	
Transformation of Graphs	• Translation of $y = f(x)$ by a units in the (+/-)ve (x/y) direction. • Scaling of $y = f(x)$ parallel to the (x/y) axis by a factor of a. • Reflection of $y = f(x)$ about the (x/y) axis.
Functions	• To prove 1-1: Show that every horizontal line $y = b$, $b \in R$ cuts the graph at most once. • For fg to exist, $R_g \subseteq D_f$.

<table>
<tr><td>Equations and Inequalities</td><td></td></tr>
<tr><td>Differentiation</td><td>$f''(x) < 0$, the curve is concave downwards; vice versa.</td></tr>
<tr><td>Integration</td><td>Integration by parts: $\int u\,dv = uv - \int v\,du$
$\int_a^b f(x)\,dx = F(b) - F(a)$
Area of region under a curve: $\int drectangle$.
Volume of rotation of a region: $\int dcylinder$.</td></tr>
<tr><td>Power Series</td><td>Maclaurin Series: $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$
Small angle approximations: $\begin{cases} \sin x \approx x \\ \cos x \approx 1 - \frac{x^2}{2} \\ \tan x \approx x \end{cases}$</td></tr>
<tr><td>Differential Equations</td><td></td></tr>
<tr><td>Sequences and Series</td><td></td></tr>
<tr><td>Complex Number</td><td>$\sqrt{zz*} = |z|$</td></tr>
<tr><td>Permutations and Combinations</td><td></td></tr>
<tr><td>Probability</td><td>To prove independence: $P(A|B) = P(A)$ Write the answer as decimals.</td></tr>
<tr><td>Discrete Random Variables</td><td>Probability distribution function: $f(x) = P(X = x)$
$\sum_{\text{all } x} f(x) = 1$
Cumulative distribution function: $F(x) = P(X \leq x)$
Expectation (mean): $\mu = E(X) = \sum_{\text{all } x} xP(X = x)$
Variance: $\sigma^2 = Var(X) = E(X^2) - [E(x)]^2$
$Var(aX) = a^2 Var(X)$
For independent observations, $E(X_1 \pm X_2) = E(X_1 \pm X_2), Var(X_1 \pm X_2) = Var(X_1 + X_2)$</td></tr>
<tr><td>Binomial Distribution</td><td>Conditions for binomial distribution: Independence, two outcomes, constant probability of success.</td></tr>
<tr><td>Normal Distribution</td><td>Standardisation: $X \sim N(\mu, \sigma^2) \rightarrow Z = \frac{X-\mu}{\sigma} \sim N(0,1)$
In GC, key in σ instead of σ^2.</td></tr>
<tr><td>Sampling</td><td>Notations:<table><tr><td></td><td>Population</td><td>Sample</td></tr><tr><td>Mean</td><td>μ</td><td>$\bar{x}$</td></tr><tr><td>Variance</td><td>σ^2</td><td>σ_x^2</td></tr></table>
Unbiased estimates of μ: $\bar{x}$</td></tr>
</table>

	• Unbiased estimates of σ^2: $s^2 = \frac{n}{n-1}\sigma_x^2$ • Distribution of $\bar{X}$: $\begin{cases} X \sim N(\mu, \sigma^2) \rightarrow \bar{X} \sim N(\mu, \frac{\sigma^2}{n}) \\ \bar{X} \sim N(\mu, \frac{\sigma^2}{n}) \text{ approximately, when } n \text{ is large } (CLT) \end{cases}$ • Write CLT in full.
Hypothesis Testing	• Presentation: Test H_0 against H_1 (state both hypotheses). Level of significance: α% (lower/upper/two-tailed). Under H_0, $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$, hence $Z = \frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}} \sim N(0, 1)$ <u>Method 1</u> Critical region: $z < a$ (taking lower-tailed as an example). Observed test statistic: $z = \frac{\bar{x}-\mu}{\sqrt{\sigma^2/n}} > a$ Hence we do not reject H_0. <u>Method 2</u> By using GC, *p*-value = *p* Since $p >$ α%, we do not reject H_0. Therefore, there is insufficient evidence at α% level of significance to claim H_1 (stated). • When using CLT, remember to convert sample variance to unbiased estimate of population variance.
Correlations and Regressions	