PC1101 Frontiers of Physics

AY2022/23 Semester 1 · Prepared by Tian Xiao

Elementary Physics

Fundamental Units of Measurement

Base Quantity	SI Unit	Derived Quantity	SI Unit
Length, l	Metre, m	Volume, V	m^3
Mass, m	Kilogram, kg	Density, ρ	$
Time, t	Second, s	Velocity, v	$
Temperature, T	Kelvin, k	Force, F	N
Electrical current, I	Ampere, A	Acceleration, a	$m\ s^{-2}$

Kinematics

Kinematics is the study of motions with respect to time t within the framework of 3-dimensional space. In a basic type of motion, where a body travels in a straight line with initial velocity u and constant or uniform acceleration a, we have:

- Final velocity, $v = u + at$ ①.
- Displacement, $s = \frac{1}{2}(u+v)t = ut + \frac{1}{2}at^2$ ②. Displacement is represented by the area under the velocity-time curve.
- From the two equations above, we have $v^2 = u^2 + 2as$ ③. Equations ①, ② and ③ are called ***Equations of Motion***.

A commonly seen motion with uniform acceleration is falling under gravity. Suppose a body falls a distance h in the first second, then the total distance s it falls after t seconds can be represented by

$$s = t^2 h$$

This equation is called the ***Law of Falling Bodies under the Influence of Gravity***. By differentiating this equation with regards to t, we get the velocity of the falling body at time t

$$v_t = \frac{ds}{dt} = 2ht$$

Dynamics

Dynamics is the study of motions with the consideration of forces. The relationship between forces and motions can be summarised by the three Newton's Laws as follows:

- [***Newton's First Law***] Every body remains stationary or moves with uniform velocity unless it is made to change this state by external forces.

- [***Newton's Second Law***] If a force acts on a body and produces a certain acceleration, then the force is proportional to the product of mass of the body and the acceleration. The acceleration takes place in the direction of the forces. Mathematically,

$$\sum F = ma$$

- [***Newton's Third Law***] If 2 bodies A and B are in contact, A will exert a force on B and B will exert an equal but opposite force on A.

Momentum

Momentum of a body p is defined as the product of its mass and velocity, i.e $p = mv$. When the velocity of a body changes, there is a change to its momentum and this quantity is called the **impulse** of the force on the body. We have:

$$\Delta p = mv - mu = F \cdot t$$

- [***Principle of Conservation of Momentum***] The total momentum within a closed system remains constant. For example, in the collision of two bodies A and B with masses m_A and m_B and velocities u_A and u_B, their final velocities after collision, v_A and v_B, should satisfy $m_A u_A + m_B u_B = m_a v_A + m_B v_B$.

Work and Energy

When a force acts to a body and causes it to move, the force is doing work on the body. Formally, if a force F moves the body through a distance s in the direction of the force, the work done W by the force satisfies $W = F \cdot s$.

The energy of a body due to its motion is called **kinetic energy**, and is defined by the function $KE = \frac{1}{2}mv^2$. If the velocity of a body changes due to a force exerting on it, then the work done by the force $W = \Delta KE$.

A body can also have energy when it is raised through a height h. This type of energy is called **gravitational potential energy** and satisfies $GPE = mgh$. Similar to kinetic energy, when the height of body changes due to a force exerting on it, then the work done by the force $W = \Delta mgh$.

- [***Principle of Conservation of Energy***] The total energy in a closed system is constant. Specifically, if we do not consider energy loss through fraction, heat, etc., then $KE + GPE$ remains constant.

Gravitational potential energy is a result of the attraction between any two masses. This force of attraction is generalised through the ***Universal Law of Gravitation***, which states that

1. Every mass attracts every other mass.
2. Attraction F_g is directly proportional to the product of their masses M_1 and M_2.
3. Attraction F_g is inversely proportional to the square of the distance r between their centers.

Mathematically,

$$F_g = G\frac{M_1 M_2}{r^2}$$

To represent the work done by the force of attraction, we define the **Newtonian Gravitational Potential** as

$$V(r) = G\frac{M_1 M_2}{r}$$

Since **force is equal to the negative derivative of potential energy**, the relationship between F_g and $V(r)$ is

$$F_g = -\frac{d}{dr}V(r)$$

Another similar inverse square law is about the attraction between two charges. When we bring a positive and a negative charge together, they will atract; when we bring two positive or two negative charges together, they will repel. The force is represented by

$$F_e = k\frac{e_1 e_2}{d^2} = -\frac{d}{dr}EPE$$

Another example of the relationship between force and potential energy is the elastic spring. According to Hooke's Law, the force needed to extend an elastic spring through a distance x is $F = -kx$, where k is some constant measuring the elasticity of the spring. Since force is the negative derivative of potential energy, the elastic potential energy here is

$$U = \int_{-x}^{x} -(-kx)\,dx = \frac{1}{2}kx^2$$

Physics Quantities

In physics, there are two type of quantities, **scalar** and **vector**. A scalar is a quantity that is described by one number, whereas a vector is a quantity that is described by more than one number.

Elementary Mathematics

Limits

- [***L'Hôpital's Rule***] Consider two functions f and g that are both differentiable. If at a point x^* we have $\lim\limits_{x\to x^*} f(x) = 0$ and $\lim\limits_{x\to x^*} g(x) = 0$, or $\lim\limits_{x\to x^*} |f(x)| = \infty$ and $\lim\limits_{x\to x^*} |g(x)| = \infty$, then $\lim\limits_{x\to x^*} \frac{f(x)}{g(x)} = \lim\limits_{x\to x^*} \frac{f'(x)}{g'(x)}$.

Differentiation

Function	x^n	$\sin x$	$\cos x$	e^x	$\ln x$	$c\,f(x)$
Derivative	nx^{n-1}	$\cos x$	$-\sin x$	e^x	$\frac{1}{x}$	$c\,f'(x)$

There are several differentiation rules:

- [***Product Rule***] $\frac{d}{dx}(uv) = u'v + uv'$.
- [***Chain Rule***] $\frac{d}{dx}(f(u)) = \frac{d}{du}(f(u)) \times \frac{du}{dx}$.
- Total differential: Suppose we have a function $f(x_1, x_2, \cdots, x_n)$. Then its total derivative with regards to t is equal to

$$\frac{df}{dt} = \left(\frac{\partial f}{\partial x_1} dx_1\right) + \left(\frac{\partial f}{\partial x_2} dx_2\right) + \cdots + \left(\frac{\partial f}{\partial x_n} dx_n\right)$$

Integration

Function	$x^n, n \neq -1$	$\frac{1}{x}$	$\sin x$	$\cos x$	$(ax+b)^n, n \neq -1$	$\frac{f'(x)}{f(x)}$	$\sin(ax+b)$	$\cos(ax+b)$
Integral	$\frac{x^{n+1}}{n+1}$	$\ln\lvert x\rvert$	$-\cos x$	$\sin x$	$\frac{(ax+b)^{n+1}}{a(n+1)}$	$\ln\lvert f(x)\rvert$	$-\frac{1}{a}\cos(ax+b)$	$\frac{1}{a}\cos(ax+b)$

There are some special integration techniques:

- [***Integration by Parts***] $\int u\, dv = uv - \int v\, du$.

Complex Numbers

The introduction of complex numbers allow us to express the identity $i = \sqrt{-1}$. A complex number can be represented on an Argand diagram with a real axis and an imaginary axis. Each complex number z has the following properties:

1. Magnitude, $|z| = \sqrt{Re(z)^2 + Im(z)^2} = \sqrt{z^* z}$.
2. Direction, $\theta = \tan^{-1} \frac{Im(z)}{Re(z)}$.
3. Conjugate, $z^* = Re(z) - Im(z)i$.

In engineering, a complex number can also be represented with its magnitude r and direction θ as

$$r\angle\theta$$

- [***Euler's Equation***] $e^{i\pi} + 1 = 0$.
- [***De Moivre's Formula***] $e^{\pm i\theta} = \cos\theta \pm i \sin\theta$. From this formula, we have $\cos x = \frac{e^{ix}+e^{-ix}}{2}$ and $\sin x = \frac{e^{ix}-e^{-ix}}{2i}$.
- Hyperpolic functions: $\cosh x = \frac{e^x+e^{-x}}{2}$ and $\sinh x = \frac{e^x-e^{-x}}{2}$.
- Derivatives and second derivatives of a complex number z: $\frac{dz}{d\theta} = iz$; $\frac{d^2z}{d\theta^2} = -z$.

Here we have a very important group of quantities, exponentials. They are commonly seen in nature such as

- [***Boltzmann's Distribution Law***] Consider a gas in equilibrium at temperature T. The probability of finding a given molecule in a given phase space cell of fixed energy E is $P = Ae^{-\frac{E}{kT}}$.
- The radioactive decay phenomenon can be described as $\frac{dN}{dt} = -\lambda N$. Equivalently, $N = Ce^{-\lambda t}$.
- In **Poisson Distribution**, $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$.
- Exponentials also occur in the famous **Schrödinger Equation**.

Waves

In physics, waves can be modelled as sine and cosine curves. A moving wave satisfies the following equations:

$$v = f\lambda$$

$$f = \frac{1}{T}$$

where v is the velocity of the wave, f is the frequency, λ is the wavelength and T is the period.

A wave with equation $y = a\sin\omega t$ satisfies $\omega = 2\pi f = \frac{2\pi}{T}$. Its accelerate is equal to $a = -\omega^2 x$ by differentiating its velocity.

Relativity

Galilean and Newtonian Relativity

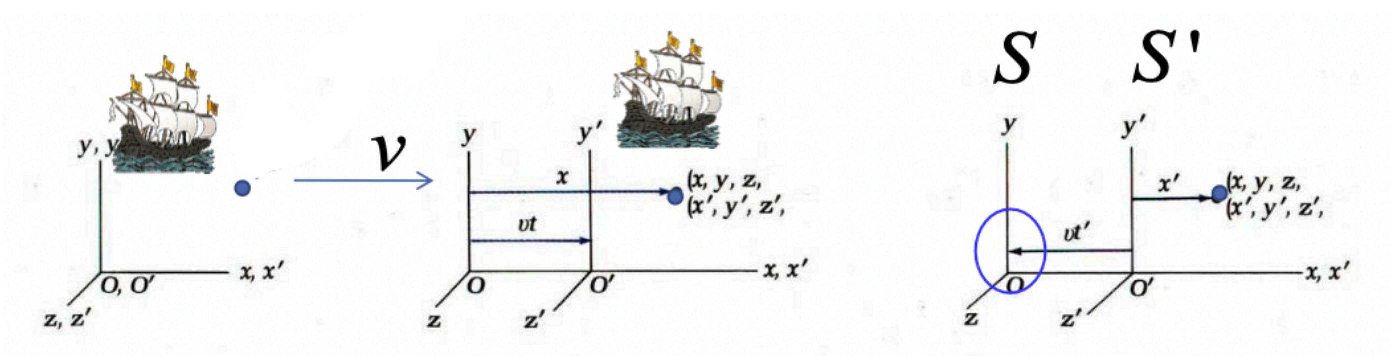
It was believed in ancient times, that force is needed to sustain motion. Galileo was the first one to realise that this reasoning is wrong. He raised the ***Law of Inertia***a, which states that a body on which forces have ceased to act keeps moving with the same speed and in the same direction as it had at the instant when these forces ceased. The reluctance to change is called **inertia**. Newton restated the same idea in his Newton's First Law, with respect to the **absolute space** which in its own nature without regard to anything external remains always similar and immovable, and **absolute time** which of itself and from its own nature flows equably without regard to anything external.

This leads us to the ***Principle of Relativity***, which states that ① the laws of mechanics in a frame of reference moving rectilinearly and uniformly through absolute space are exactly the same as in another frame which is at rest in absolute space. These frames are called **inertial frame of reference**, where no forces are acting on it such that it has a constant velocity. Principle of Relativity in its second form states that consider O and O' to be observers with O at rest (absolute space) and O' moving with a uniform velocity and there is an object in the absolute space which is acted on by a force. Then O would describe it by saying $F = ma$ and O' would say $F' = ma'$. Principle of Relativity in its third form states that ③ the laws of mechanics remain the same for observers in inertial frames that are in uniform motion with respect to each other.

Galilean Relativity

Say observer O is in the S frame and observer O' is in the S' frame. Both S and S' are inertial frames and S' is moving at a velocity v with respect to S. Consider an object at (x, y, z) at time t observed by O. Under O''s observation, the position of the object would be

$$\begin{cases} x' = x - vt \\ y' = y \\ z' = z \\ t' = t & \text{assuming clocks behave in the same way in } S \text{ and } S' \end{cases}$$



(a) (b) (c)

The consequences of these transformations, called **Galilean Relations**, are as follows:

- [***Law of Addition of Velocities***] $u' = u - v$. Hence $a' = \frac{du'}{dt} = \frac{du}{dt} = a$, meaning that $F = ma$ still holds. Newton's Second Law obeys Galilean Relations.

Before Einstein's Relativity, people consider relativity under Galilean Transformations. It was believed that even under transformations, some physics quantities should be **invariant** (it does not change) and physic principles should be **covariant** (the formula does not change). It turns out that Maxwell's Equations get changed under Galilean transformations, so it is not covariant. This means

1. The principle of relativity does not apply to electromagnetic waves. OR
2. Maxwell's Equations are wrong. OR
3. Galilean Transformations are wrong.

Before Einstein's Relativity, people also believed that when there is no air, light transmits in some medium called **ether** in the absolute frame. The Michelson-Morley Experiment proves that there is no either.

Einstein's Special Relativity

Einstein believed that the Laws of Physics should apply in the same fashion everywhere, so he tried to find a new transformation that may preserve the Principle of Relativity and the Speed of Light, c. His axioms are:

1. The Laws of Physics are covariant in all inertial frames.
2. [***Principle of Constancy of Speed of Light***] The Speed of Light in free space has the same value c in all intertial systems.

Einstein believed that the transformation between the two frames must be linear, in order to ensure that Newton's Laws hold in all inertial frames. Hence, we can write the coordinates of an event in one frame as linear combinations of the coordinates in the other frame.

Einsteinian Transformation

Say observer O is in the S frame and observer O' is in the S' frame. Both S and S' are inertial frames and S' is moving at a velocity v with respect to S. Consider an object at (x, y, z) at time t observed by O. Under O''s observation, the position of the object would be

$$\begin{cases} x' = (x - vt)\gamma \\ y' = y \\ z' = z \\ t' = (t - \frac{vx}{c^2})\gamma \end{cases}$$

The term $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1$ is called the **Lorentz Factor** and this transformation is also called Lorentz Transformation. In matrix form, we have

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma \frac{v}{c^2} & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

We notice that $t' \neq t$, meaning that moving clocks run differently as compared with stationary clocks. It also depends on its location. We also notice that the Lorentz Factor ensures that $v < c$.

This transformation preserves both Newton's Laws and Maxwell's Equations, hence satisfies Einstein's axioms.

Reversely, if observer O' observes that an object is at (x', y', z', t'), then the attributes observed by observe O would be the ***Inverse Lorentz Transformation***:

$$\begin{cases} x = (x' + vt)\gamma \\ y = y' \\ z = z' \\ t = (t' + \frac{vx}{c^2})\gamma \end{cases}$$

If we draw the space-time diagram of S' frame, the x'-t' axes will be skewed because Lorentz Transformations are not orthogonal.

This transformation has several consequences different from Galilean Relations:

1. **Velocity Addition**. $u = \frac{u'+v}{1+\frac{u'v}{c^2}}$. This implies that the Speed of Light is always constant.
2. **Simultaneity**. Consider two events at (x_1, t_1) and (x_2, t_2). In the S' frame, we have

   $$t'_2 - t'_1 = \frac{v}{c^2}(x_1 - x_2)\gamma$$

   which means that simultaneous events in one frame are not simultaneous in moving frames.
3. **Lorentz Contraction**. $L_0 = L\gamma$ or $L = \frac{L_0}{\gamma}$. Lengths observed by observers in different frames are not the same. Specifically, L_0 is called the **proper length** measured by the person at rest with the object.
4. **Time Dilation**. $T = T_0\gamma$. Time ticks more slowly in moving frames.
5. **Momentum**. $p = \gamma m_0 v$.
6. **Mass**. $m = \gamma m_0$.
7. **Kinetic Energy**. $KE = mc^2 - m_0c^2 = (\gamma - 1)m_0c^2$. Here $E = mc^2$ represents the energy or the mass of a body.
8. **Total Energy**. Total energy equals the sum of rest enerfy and momentum of any moving particle: $E^2 = (m_0c^2)^2 + (pc)^2$.

Einstein's General Relativity

In mathematical physics, **Minkowski Space** is a combination of three-dimsensional Euclidean space and time into a four-dimensional manifold where the space-time interval between any two events is independent of the inertical frame of reference in which they are recorded. In all frames of reference, (x, y, z, t) satisfies $-c^2t^2 + x^2 + y^2 + z^2 = 0$. According to Einstein's summation convention,

$$ds^2 = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} dx_\mu dx_\nu = g_{\mu\nu} dx_\mu dx_\nu$$

In the last term, $g_{\mu\nu}$ is to a matrix. It has some special cases:

1. If motion of any body not travelling at the speeds close to c and in the vicinity of low intensity gravitational field, this is a generalisation of the 3-dimensional distance relation to 4 dimensions.
2. [***Einstein's Field Equation***] $R_{ij} - \frac{1}{2}Rg_{ij} = \frac{8\pi G T_{ij}}{c^4}$. Here T_{ij} contains all forms of energy and momentum. Hence in an empty space time equation where no matter and energy is present, we have $R_{ij} = 0$.

There is no force concept in Einstein's General Relativity. Instead, he thinks space determines how mass should move and mass determines how space should curve.

Einstein made a statement about the equality of intertial mass and gravitational mass:

$$\text{Inertial mass} \times \text{Acceleration} = \text{Intensity of gravitational field} \times \text{Gravitational mass}$$

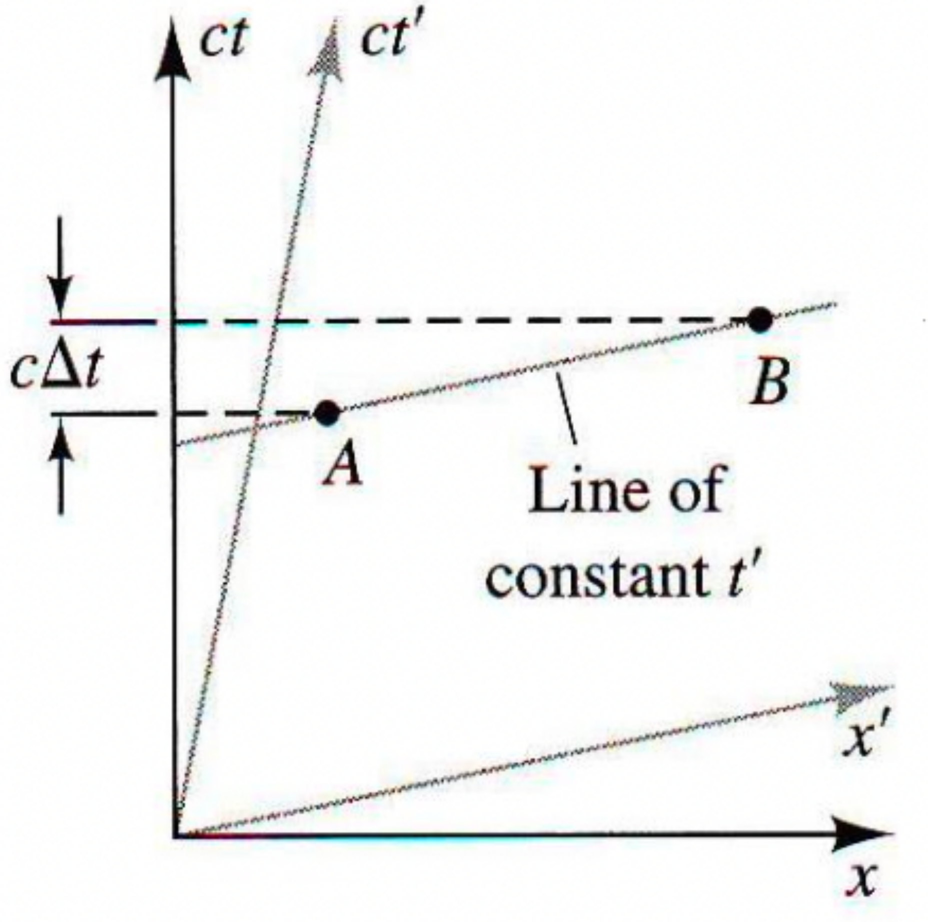
This suggests that only when inertial mass and gravitational mass are equal, the acceleration is independent of the nature of the body. As a result, he stated the ***Equivalence Principle*** as all objects fall at the same rate, assuming negligible air resistance. Considering a spaceship far away from gravitational influence in uniform acceleration with respect to distant stars. When an object is thrown horizontally, an observer outside the spaceship observes a straight line path of the object, but the observer inside the spaceship observes a curved parabola path. Einstein argues that the same holds true for a beam of light.

Equivalence Principle has the following consequences:

1. Gravitation causes time to slow down.
2. Gravitation causes space to be non-Euclidean but rather Riemannian.
3. When an object moves, the surrounding warp of space and time moves to readjust to a new position. This readjustments produce ripples in the overall geometry of space-time, called **gravitational waves**.

Space-Time Diagrams

The x'-t' diagram is skewed because $\frac{\Delta x}{\Delta t} = \frac{\Delta x'}{\Delta t'} = c$. This also demonstrates why there is time dilation and length contraction:



Therefore, time dilation is always accompanied by length contraction.

In a skewed space-time diagram, the axes are not orthogonal to each other. Suppose

$$x = x(x_1, x_2, x_3); y = y(x_1, x_2, x_3); z = z(x_1, x_2, x_3)$$

Then in the new coordinates, by total differentiation we have

$$dr^2 = dx^2 + dy^2 + dz^2 = \sum_{\mu=1}^{3}\sum_{\nu=1}^{3} g_{\mu\nu} dx_\mu dx_\nu$$

Intuitively, in a right angle triangle, $c^2 = a^2 + b^2$, but in other triangles, $c^2 = a^2 + b^2 - 2ab\cos\theta$.

Black Holes

In the 1700's, John Michell and Pierre Laplace both asked the possibility of an astronomical object whose mass and radius satisfy the criteria $R = \frac{2Gm}{c^2}$, then even light cannot escape. Under Newtonian Relativity, the velocity required to escape from a planet is equal to $v = \sqrt{\frac{2Gm}{R}}$.

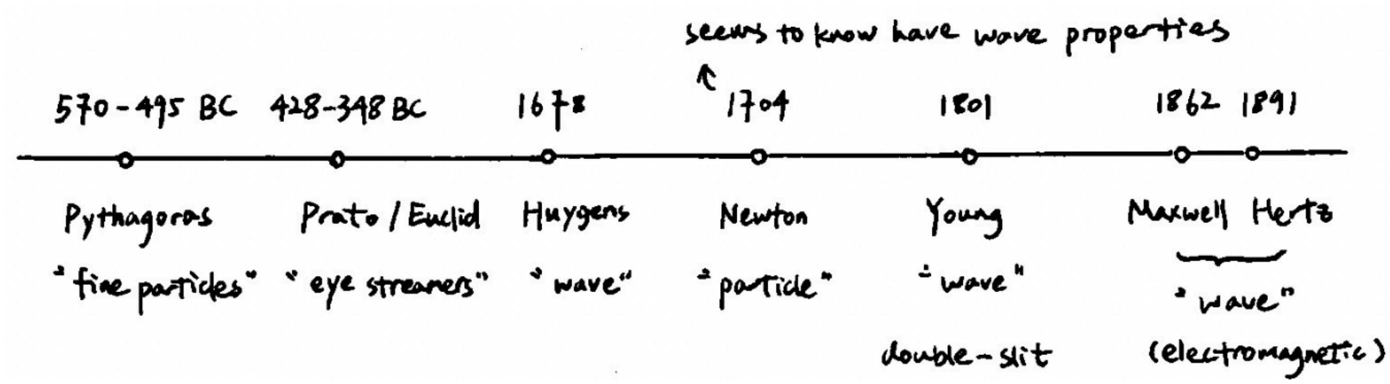
Consider B on the surface of a very massive star. B sends out signals every second to A, but the star starts to collapse. As the star collapses, B experiences an increasing force of gravity. Gravitational red shift comes into effect because time slows down. Eventually, the dying star will approach a boundary, called **Schwarzschild Radius** where A will have to wait infinitely long for the next signal from B. Therefore, when B crosses the sphere of Schwarzschild radius $R = R_s = \frac{2Gm}{c^2}$, no signals from B will ever reach A. R_s is called the event horizon, because no events taking place within the event horizon will ever be observed by an outside observer like A. No mass can escape from a black hole not even light.

Quantum Physics

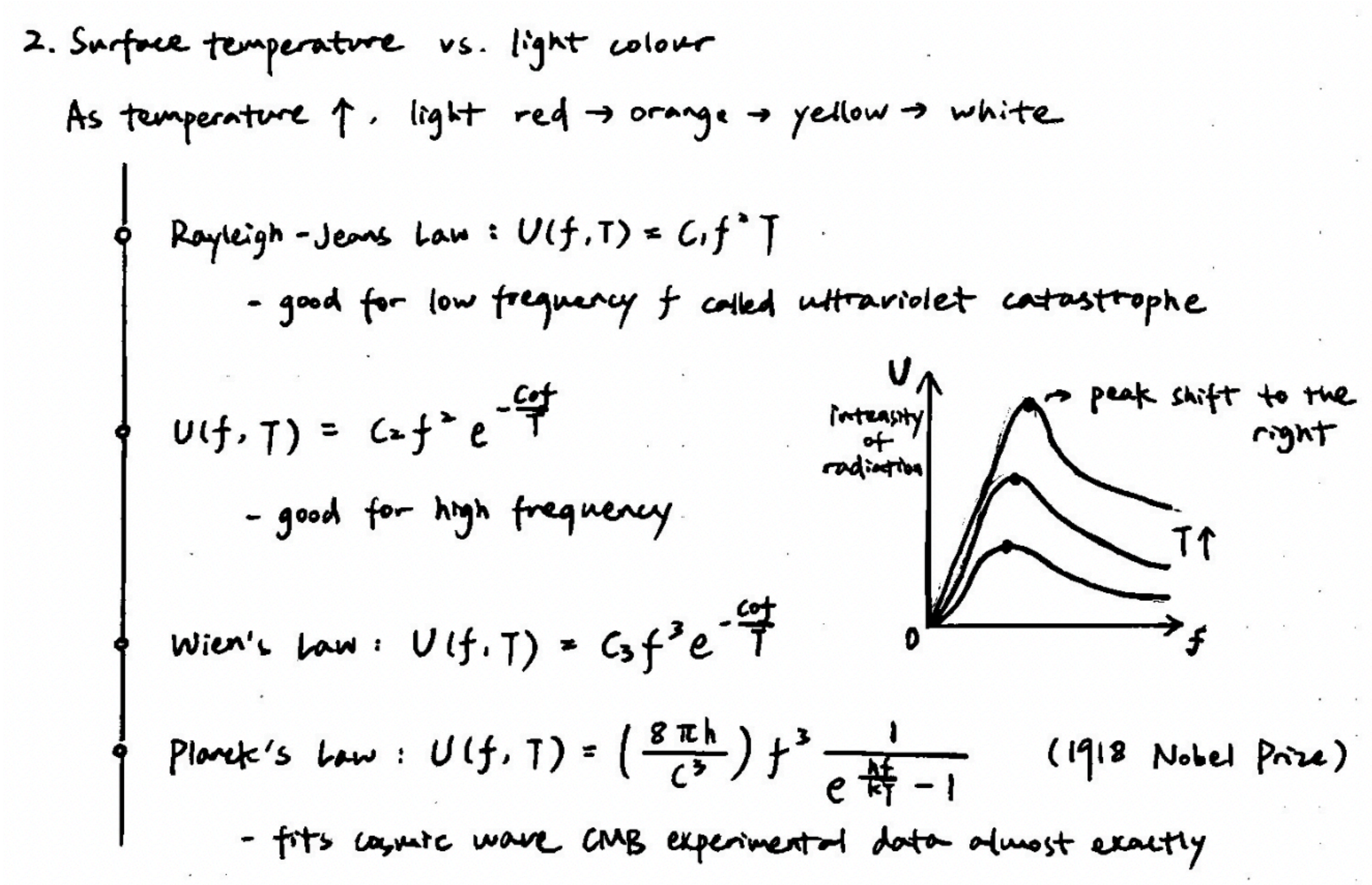
From Classical Mechanics to Quantum Mechanics

Perception of Light

- Pythagoras believed that light emanates from luminous bodies in the form of very fine particles;
- Plato and Euclid believed that light consists of streamers emitted by the eye;
- Huygens advocated a wave theory of light;
- Newton believed that light is a strem of particles of corpuscles. However, he seems to also know that light particles have certain wave properties;
- Young did the double slit experiment and finally showed that light is a wave phenomenon;
- Maxwell and Hertz vertified that light is a electromagnetic wave.

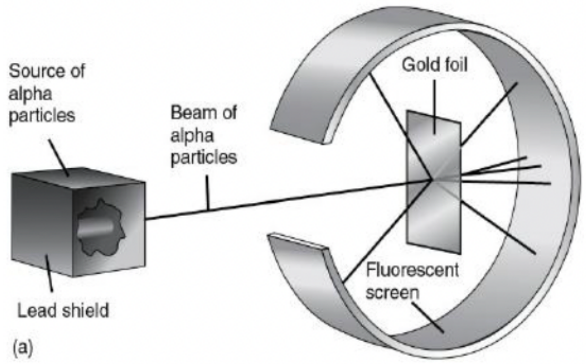


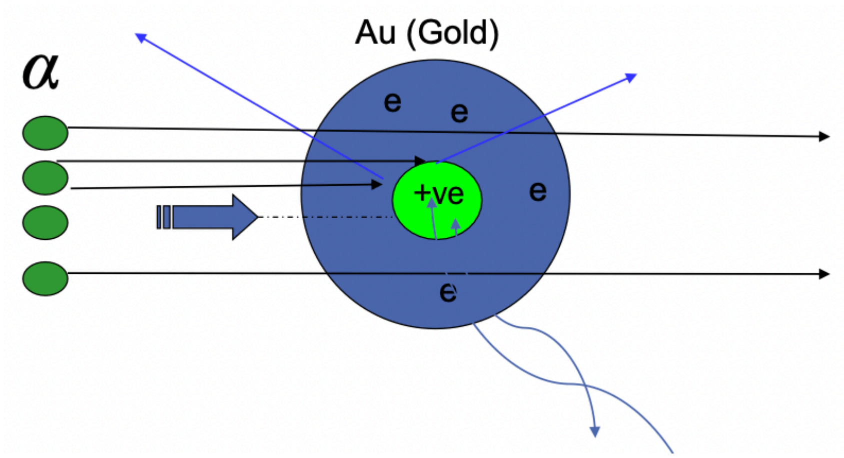
Temperature and Light Colour



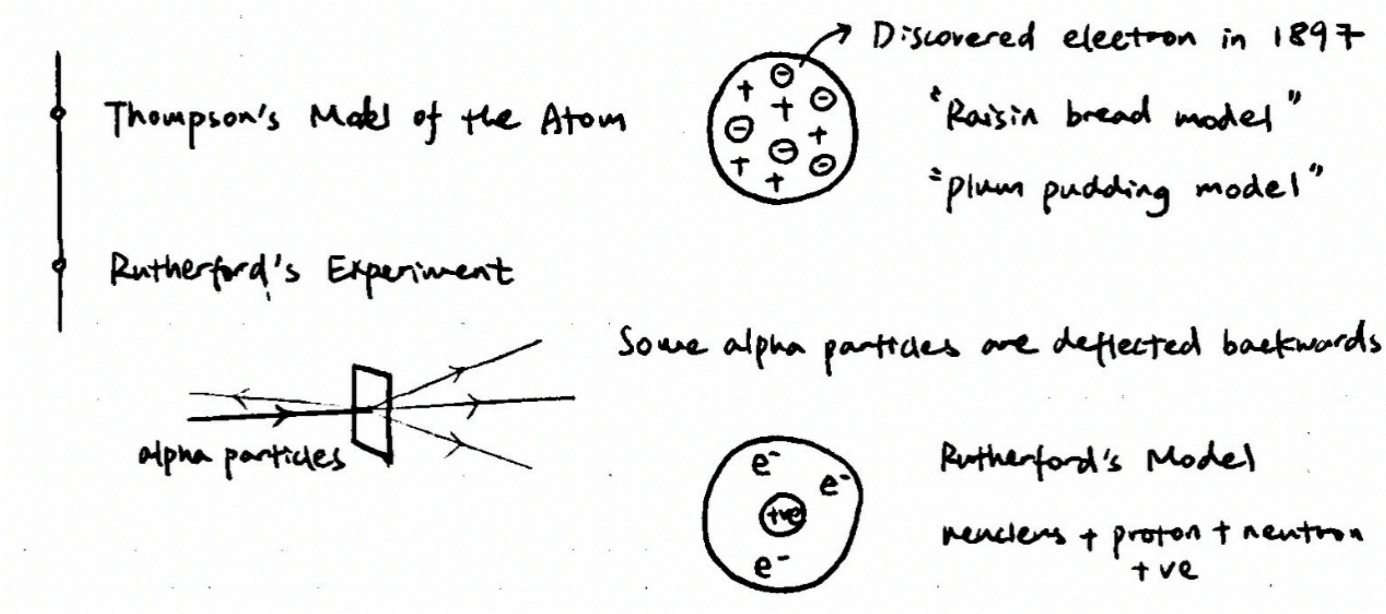
Discovery of Nucleus

Thomson discovered the electron in 1897 and described the atom as a spherical object containing N electrons confined in homogenous jelly-like but relatively massive positive charge distribution whose total charge cancels that of the N electrons. Rutherford tried to vertify the model in 1911 but found something strange - some alpha particles are deflected almost backwards.





Therefore, Rutherford discovered the Nucleus where positive charges reside and identified the positive charges as protons. He also theorized the existence of neutrons in the nucleus.



Photoelectric Effect

Hertz discovered in 1887 that electrons can be ejected when one shines light on metal. He also had the following observations:

1. The rate at which electrons are ejected is proportional to the brightness (intensity) of the light.
2. The maximum energy of the ejected elctrons is not affected by the brightness (intensity) of the light.
3. The electron's maximum kinetic energy depends on the frequency of light.
4. The time lag between turning on the light and the ejection of the first electron is not affected by the brightness (intensity) or frequence of the light.

5. The effect is easy to observe with violet or ultraviolet light but not with red light.

These phenomena can be concluded as $E_{\text{electron}} \propto f \propto \frac{1}{\lambda}$. Einstein interpreted this effect as $E_k = hf - \phi$. The hf absorption by electron is an all or nothing process and is immediate, so there is no delay as wave energies build up. hf is called a quantum unit or quanta.

To sum up, photoelectric effect is where electrons are emitted when light photons interact with atoms of metal. The number of photons in a light beam controls the brightness of the whole light beam, whereas the frequency of the light controls the energy of each individual photon. The photoelectric effect shows conclusively that light being a wave has particle-like properties ($E_{\text{light particle}} = hf$).

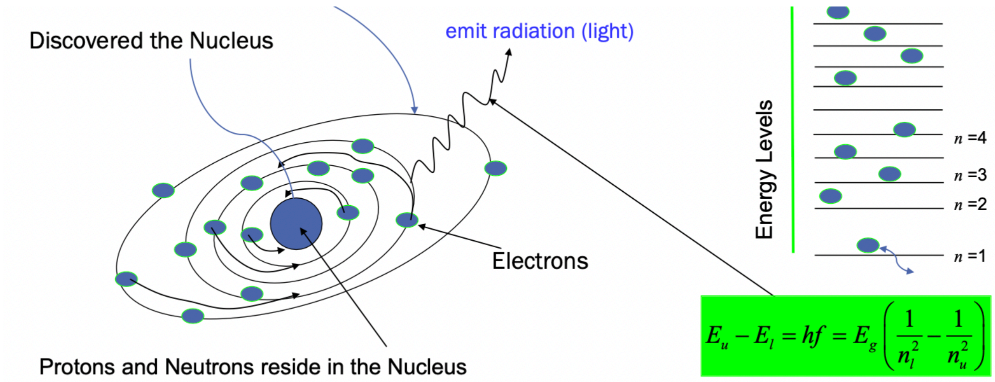
Compton Effect

Many scientists including Compton, Lorentz, Bohr were still skeptical of the particle-like behaviour of light. Compton asked: If light did consist of quanta as Einstein said, could one bounce them off each other? Compton then observed that the scattered photon has a less energy than the incident photon, which means that the frequence is lowered. He proved that light behaves as if it consists of discrete energy projectiles, not only in regard to energy transfer but also in regard to momentum transfer.

Quantum Theory

Wave-Particle Duality and Bohr's Model

Einstein's photon hypothesis in 1905 stated that light being a wave behaves like a particle ($E = hf$). **In 1924, de Broglie stated that perhaps other small micro particle can behave like waves, such as electrons ($p = \frac{h}{\lambda}$).** Bohr tried to make sense of these ideas and stated that if an electron is a wave, say in the hydrogen atom, it would have been reasonable to use the circumference of the circular orbit to be the wavelength ($\lambda = 2\pi r$), hence he came up with the Bohr Quantization condition $L = rp = \frac{h}{2\pi}n$, where h is the angular momentum, p is the ordinary linear momentum. Bohr's mini solar model is able to explain where light comes from the excites gas: electron jumps between upper levels and lower levels ($E_u - E_l = hf = E_g(\frac{1}{n_l^2} - \frac{1}{n_u^2})$). Bohr was inspired by Balmer who arranged the spectra lines of hygrogen into groups and write down a neat empirical formulae for wavelengths (Balmer series: $f_n = \frac{4c}{k}(\frac{1}{n_l^2} - \frac{1}{n_u^2})$).



There are also experimental evidence for energy levels. Franck-Hertz experiment in 1925 was the first experiment to provide support for the discrete energy levels.

Bohr's model's problem: Why does the electron stay in the excited energy level for an unspecified period before it descends to a lower energy level ? What is the physical mechanism that makes it wait?

Classical physics: A round stone sits on the side of a hill, popped by another stone, and the second is then removed, the round stone will roll down hill, immediately. It is because of the action of the force of gravity that is there all the time.

In comparison, the electron (-ve) in the excited state of the hydrogen atom does not drop to the lower energy level because of the attractive electric force of the nucleus (+ve) on it.

However, after the energy drops but before it drops to E_{low} it will have lose energy but there is no photon to take the energy yet.

Bohr wants to give up.

Heisenberg Uncertainty Theory

Heisenberg thought that one should try to construct a theory in terms of quantities which are provided by experiment, rather than to build it up on the basis of a model which involves many quantities which cannot be observed. Hence, an atom would be represented not by a physical picture, but by a purely mathematical model. Instead of thinking about electron orbits, one can think of the atomic model as a matrix with rows and columns of empty spaces. Each space would be filled by a number as we obtain the necessary information about that particular atom. Applying one matrix to another in a certain defined way would give a set of numbers representing the results of experimental observation. For example, applying the frequency matrix to the hydrogen matrix would give the observed values of the frequencies of ight emitted by hydrogen.

Heisenberg soon noticed something in his scheme which worried him - matrix multiplication does not obey Commutative Law. He then found that the matrices representing the position $[q]$ and momentum $[p]$ of a particle such as an electron or a proton must obey a certain commutation condition - $[q][p]-[p][q]=\frac{ih}{2\pi}[I]$, which is the famous Heisenberg Uncertainty Principle. Here h is the Planck's constant. This theory shows that the order of measurement will affect the results.

Schrodinger Theory

Schrodinger was not a fan of Hensenberg's matrix works. He thought it was too difficult. He was inspired by the following principles:

1. [***De Broglie's Idea***] $p=\frac{h}{\lambda}$. This applies to only free electrons. Schrodinger wanted to adapt it to elctrons subject to forces and particularly electrons in an atom such as hydrogen.
2. [***Principle of Least Time of Light***] Out of all possible paths that it might take to get from one point to another, light takes the path which requires the shortest time. - Feynman "Path of a lazy man." - Occam's Razor
3. [***Hamilton (Minimum) Principle***] Nature will choose a path for which the action $S=\int_{t_1}^{t_2}(KE-PE)dt$ is a minimum.
4. [***Fermat's Principle***] Out of all possible paths, light takes the path which requires the shortest time.

Schrodinger came up with the famous Schrodinger Wave equation: $\nabla^2\psi(r)=-\frac{8\pi^2m}{h^2}(E-V)\psi(r)$. In one dimension (string of length L), the solution is a wave $\phi(x)=A\sin kx$ with $k=\frac{2\pi n}{L}, n=1,2,3,\cdots$.

[***Time Independent Schrodinger Equation in One Dimension***] $-\frac{\hbar}{2m}\frac{d^2\Psi(x,t)}{dx^2}=(E-V)Psi(x)$

[***Time Dependent Schrodinger Equation***] $\frac{\hbar}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}=-i\hbar\frac{\partial\Psi(x,t)}{\partial t}+V\Psi(x,t)$. Note that $E=-i\hbar\frac{\partial}{\partial t}$.

What does Schrodinger Equation tell us? Essentially a wave equation, the Schrödinger equation describes the form of the probability waves (or wave functions [see de Broglie wave]) that govern the motion of small particles, and it specifies how these waves are altered by external influences.

Maxwell wave equations

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

3 Dim. Reminder:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Ordinary wave equation

$$\nabla^2 \vec{w} = \frac{1}{v^2} \frac{\partial^2 \vec{w}}{\partial t^2}$$

Schrodinger wave equation (1 dimension)

$$\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} = -i\hbar \frac{\partial \Psi(x,t)}{\partial t} + V\Psi(x,t)$$

A trial solution, the Plane Wave Ansatz, is $\Psi(x,t) = Ae^{i(kx-\omega t)}$. Ψ is sometimes called State function or wave function.

Given the successful results obtained in his first paper and the duality of matter stressed by de Broglie, Schrodinger was now convinced that **one should give up** "ideas about the place of the electron and path of the electron". If these are not given up, contradictions remain".

G. Venkataraman

Wave

Einstein also thought de Broglie was audacious. To Born, Einstein said Although it seems to have been written by a madman, it has been written soundly.

These young men consider it exceedingly easy to discard old concepts in physics.

Lorentz

Function

Wave function leads to the uncertainty principle: $\Delta x \propto \frac{1}{\Delta f}$ (Bandwidth theorem). $\Delta x \Delta p \approx \frac{h}{2\pi} \approx \hbar$.

In the case of measuring the microscopic objects like electrons, even a single photon bouncing off an electron can appreciably alter the position of the electron and in an unpredictable way. The significance of the uncertainty principle lies in that even in the best of conditions, the lower limit of uncertainty is $\overline{h}$, i.e. if we wish to know the momentum of an electron with great accuracy (Δp) the corresponding uncertainty in position Δx will be large or vice versa ($\Delta x \Delta p \approx \hbar$). We cannot measure a particle energy with complete precision in an infinitesimally short span of time ($\Delta E \Delta t \approx \hbar$). It just means that we are more uncertain of the time during which the particle of investigation has that energy.

"We can easily forgive a child who is afraid of the dard; the real tragedy of life is when men are afraid of the light." Plato said that one can never understand the universe until the smallest components of matter are known. Heisenberg was influenced by this tenet of Plato all his life. He said by getting to smaller and smaller units, we do not come to fundamental or indivisible units. But we do come to a point where further division has no meaning.

$|\Psi|^2$

Born wrote in a footnote "a more precise consideration shows that the probability is proportional to the square of Ψ". $|\Psi|^2$ has the meaning of probability. Ψ is also called probability amplitude. We give up the mini solar system, instead we find that the electron probability cloud forms beautiful and intricate patterns. Depending on the values of n, l, m in $|\Psi_{n,l,m}|^2$, the probability of finding the electron is more in some directions/places and less in others.

Faraday did the splitting experiment to see if blurred spectra are affected by magnetic fields. In 1896, Zeeman from Holland saw the broadened (blurred) lines. Lorentz explained it using physics of that time. But when spin is added, anomalous Zeeman effect cannot be explained by classical physics.

Uhlenbeck and Goudsmit studies the problem very carefully and also came up with the idea of electron spin but with a twist - it is not physically spinning but possess a mysterious spin (angular momentum). Spin is just a label. Now we need 4 labels. The spin did not come from solving the Schrodinger equation. It is a special kind of spin (intrinsic spin).

Given the successful results obtained in his first paper and the duality of matter stressed by de Broglie, Schrodinger was now convinced that one should give up ideas about the place of the electron anbd path of the electron. Einstein also thought de Broglie was audacious. To Born, Einstein said although it seems to have been written by a madman, it has been written soundly. Lorentz said that these young men consider it exceedingly easy to discard old concepts in physics.

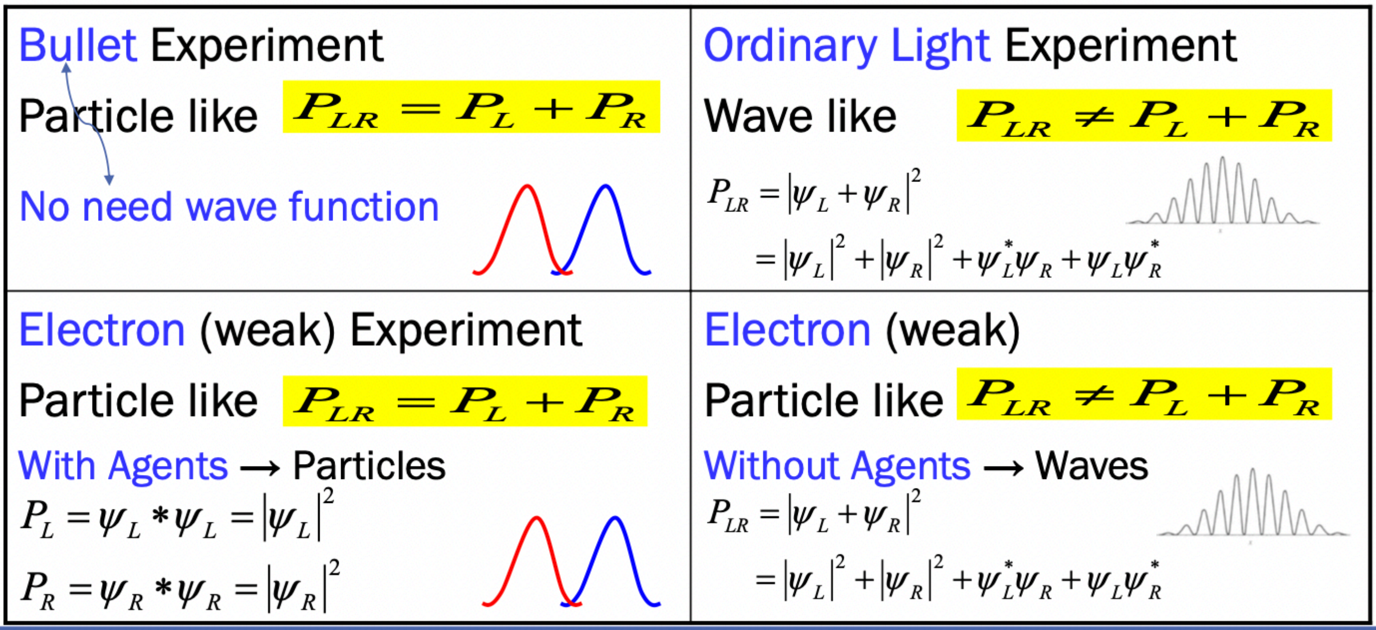
One application is the quantum tunneling. Electrons quantum tunnel in enzymes allows certain chemical processes to speed up by several orders of magnitude.

Young's Double Slit Experiment

The electron behaves like a particle when only one slit is open and behaves like a wave when both slits are open. - wave-particle duality

Light is both wave and particle, depending on what you want to see.

$P_L = |\Psi_L|^2$; $P_R = |\Psi_R|^2$; $P_{LR} = |\Psi_L + \Psi_R|^2 \neq P_L + P_R$



Heisenberg states that atoms or elementary particles are not real; they form a world of possibilities than one of the things or facts. Observations not only disturb what has to be measured but in fact produce it. We ourselves produce the results of the measurement. Actual state of existence depends on how we observe and what we choose to observe. Prior to a measurement we cannot even think that the electron has definite momentum and a definite cooridinate.

Einstein, Podolsky and Rosen (EPR) Problem

In 1934, EPR wrote a paper: can quantum mechanical description of physical reality be considered complete? The main idea is htat quantum mechanics is at best a successful theory but not a complete one. EPR's criterion for reality is that if, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physicla quantity. Since qunatum mechanics does not allow simultaneous elements of reality to be measured together, this contradicts EPR hence quantum physics cannot be claimed to be a complete thoery.

ERP concluded that the wave function (Ψ) does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. Till the end, Einstein still believes that mankind should be able to construct a classical theory which besides being deterministic, would provide a complete description of physical reality - objective reality (ontology).

Bohr wrote a rebuttal paper with the same title. He said the opposite of a deep truth is also a deep truth. Truth and clarity are complementary. Einstein commented that Bohr though very clearly, wrote obscurely and regard himself as a prophet. Bohr argued that if no observations are made, a particle cannot ahve physical real attributes.

Quantum mechanics theory is a candidate of epistemology. Hidden variable theory is a candidate for ontology. Einstein said that I must seem like an ostrich who forever buries his head in the relativistic sand in order not to face the evil quanta.

Summary

Classical Physics

Continuously quantities : e.g. Energy, momentum

Deterministic and gives rise to Causality (Theory of cause and effect)

The system already has a definite value for the property being measured even before the measurement is actually made.
The instrument or apparatus or observer does not affect the system.

Example: measuring the length of the table (macro system) and the table is not affected.

Classical Physics can stand all by itself.

Quantum Physics

Discrete quantities : e.g. Energy, momentum

Not deterministic (indeterminism) and we can only use **probabilistic ideas.**

Prior to a measurement we **cannot even think that the electron has a definite say momentum and a definite co-ordinate.**
The instrument or apparatus or observer **does affect** the system.

Example: **measuring or observing** a micro system **will disturb it.**

But Quantum Physics **needs classical physics** as a foundation.

Treatment of Experimental Data

Sample standard deviation $s = \sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}.$

Degrees of freedom = sampe size - number of constraints involved in the calculation of the statistical parameters.

Var = $\overline{x^2} - \overline{x}^2$.

Random error: When a measurement is made more than ounce, the result scatter about the average.

Systematic error: The error do not scatter around a mean value due to uncalibarated instrumentation, human reaction times, etc.

Propogated error: If $R = x \pm y$, then $\Delta R = \Delta x + \Delta y$. If $R = x \times \text{ or } /y$, then $\Delta R/R = \Delta x/x + \Delta y/y$.

Summary : 4 methods

$$D = \frac{M}{V}$$

$D(M, V)$

1)JC/Poly/IB School Method

$$\frac{\Delta D}{D} = \frac{\Delta M}{M} + \frac{\Delta V}{V}$$

2) Logarithmic Method & absolute value

$$\frac{1}{D} dD = \left|\frac{1}{M}\right| dM + \left|-\frac{1}{V}\right| dV$$

3) Total Differential Method

$$dD = \frac{\partial D}{\partial M} dM + \frac{\partial D}{\partial V} dV = V^{-1} dM + MV^{-2} dV$$

4)If uncertainties are independent and random, we use the Quadrature Method

$$(\Delta D)^2 = \left(\frac{\partial D}{\partial M}\right)^2 (\Delta M)^2 + \left(\frac{\partial D}{\partial V}\right)^2 (\Delta V)^2$$

Particle Physics

Fundamental Particles of Matter

All forms of matter are formed with only 12 particles as follows:

1. The Lepton table.

Name	Symbol	Mass	Electric Charge
Electron	e^-	1	-1
Electron Neutrino	v_e	< 0.00012	0
Muon	μ^-	207	-1
Muon Neutrino	v_μ	< 1.1	0
Tauon	τ^-	3491	-1
Tau Neutrino	v_τ	< 500	0

2. The Quark table.

Name	Symbol	Mass	Electric Charge
Up	u	2	$+\frac{2}{3}$
Down	d	6	$-\frac{1}{3}$
Strange	s	200	$-\frac{1}{3}$
Charm	c	3000	$+\frac{2}{3}$
Bottom/Beauty	b	9000	$-\frac{1}{3}$
Top/Truth	t	$?$	$+\frac{2}{3}$

Each proton is made up of two *up* quarks and one *down* quark, and hence has charge $1e$; each neutron is made up of one *up* quark and two *down* quarks, and hence has charge $0e$.

Unlike quarks, leptons may exist as separate individual objects. Electron is a stable object. Muon and tauon are allowed to decay into particles. Leptons are accompanied by electrically neural neutrinos. Fortunately, neutrinos interact with matther so rarely that even a huge number passing through us does no harm to us (around 10^{12} electron neutrinos pass through our bodies every second). Neutrinos have extrmely small but non zero mass. Our current theories do not predict the mass of neutrinos.

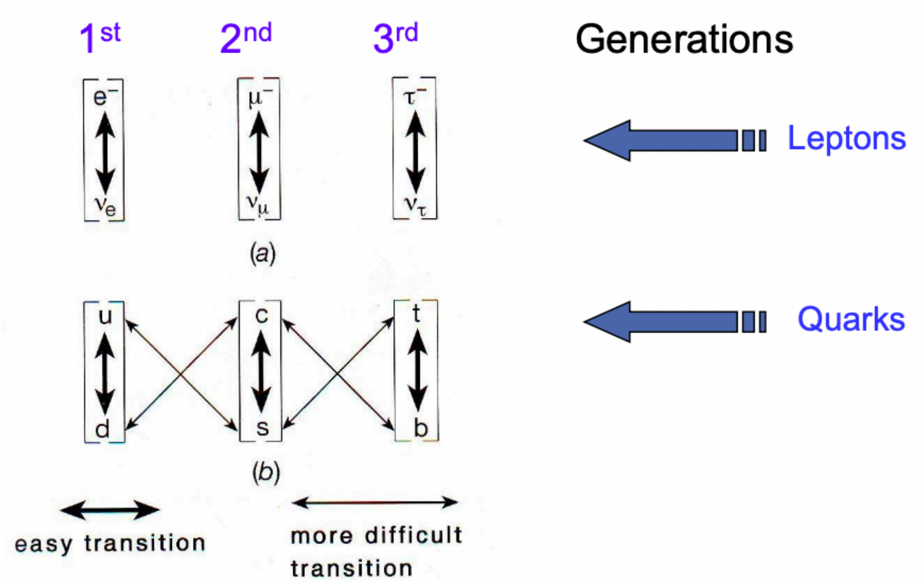
1^{st} **Generation**	2^{nd} **Generation**	3^{rd} **Generation**
Electrons are found in atoms, electric currents and radioactivity.	Muons are produced at upper atmosphere by cosmic rays.	Tauons are so far only in labs and not naturally occuring.
Electron neutrinos are found in radioactivity, atomic reactors and nucler reaction in the Sun.	Muon neutrinos are produced by atomic reactors or at upper atmosphere by comic rays.	Tauon neutrinos are so far only in labs and not naturally occurring.

There are equal number of quarks and leptons. *Strange*, *charm*, *bottom* and *top* quarks were discovered in High Energy Labs (Colliders).

Fundamental Forces

There are 4 fundamental forces:

1. Gavitational force (1686): Applies in long range.
2. Electromagnetic force (1864): Applies in long range.
3. Strong nuclear force: Applies in short range ($\approx 10^{-15}m$). Strong force only acts between quarks (e.g. to form protons and neutons). Quarks feel the strong force, while leptons do not. This means that quarks can only bind together to form particles.
4. Weak nuclear force (1968): Applies in short range ($\approx 10^{-17}m$). Instead of attracting and repelling, the weak force changes particles from one type to another. Below describes how the weak force changes particles.



As a reference, the diameter of an atom is about $\approx 10^{-10}m$. Both quarks and leptons feel 1, 2 and 4. The leptons do not bind together to form particles.

Colliders

How do we know so much of micro-stuff? In 1911, Rutherford discovered the existence of nucleus from his experiment and theorised the existence of neutrons. Recently, high energy particle (HEP) physicists have invented big machines, colliders, to observe micro-particles. There are 3 groups of HEP particle physicists - theorists, phenomenologists and experimentalists.

Following two billion dollars and six years of work, the promising Superconducting Super Collider (SCSC/SSC) project was terminated in 1993. Ring circumference was 87.1 km with an energy of 20 TeV per proton. Currently, the most powerful collider is the Large Hadron Collider (LHC) with 27 km circumference ring with 11000 times per second, total maximum energy of 14 TeV, serveral thousand billion protons each with the energy of a fly 99.9999991% of light speed and a billion collisions a second. The collider is built to study the origin of mass, the nature of dark matter, the primordial plasma, and the difference between matter versus antimatter. It has the following detectors:

Detector	Full Name	Mission
ATLAS	A Toroidal LHC Apparatus	General purpose (including searching for the Higgs boson and **S**upersymmetry) e.g. **S**quarks, **S**electron.
CMS	Compact Muon Solenoid	General purpose (including searching for the Higgs boson and supersymmetry).
ALICE	A Large Ion Collider Experiment	Creating quark-gluon plasma.
LHCb	Large Hadron Collider Beauty	Searching for CP violation in B particle decay.
LHCf	Large Hadron Collider Forward	Testing cosmic ray detection devices.
TO TEM	TOTal Elastic and diffractive cross selection Meaurement	High precision measurements of protons.

The Standard Model

The Standard Model (SM) states that forces or interactions are also particles.

Forces	Messenger Particles (Bosons with integer spins)	Matter (Fermions with $\frac{1}{2}$ integer spins)
Gravity	Graviton (massless bosons)	All matter that have mass (no charges requirement)
Electromagnetic	Photon (massless bosons)	All charged matter (+ve or -ve)
Strong	Gluons (massless bosons with 3 colours R, G, B)	Quarks, 3 types of charges (u, d, s, c, t, b (R, G, B))
Weak	W^+, W^-, Z^0 (3 massive bosons)	Quarks, 6 flavours; Leptons, 6 flavours

Neutrino Reaction

Neutrinos are real and confirmed. Wolfgang Pauli postulated it in 1932 and it was confirmed in 1950 in atomic reactors. Also, muon is established in 1950 and surfaced in 1929 in cosmic ray experiments. However, nobody is able to tell if the muon-neutrino and the electron-neutrino are different particles.

Consider the characteris reaction $X_{\text{unknown}} + n \to p + e^-$. Inside the neutron, the following fundamental reaction has taken place: $X_{\text{unknown}} + d \to u + e^-$. This is the basic reaction that has been triggered by the weak force. Note that it only involves one of the quarks within the original neutron while the other 2 quarks are unaffected. For example, considering the reaction $v_e + n \to p + e^-$, a big tank of Chlorine $^{37}_{17}Cl$ will be transformed into $^{37}_{18}Ar$ as the electron produced by the reaction would be moving quickly to be captured by the Argon nucleus.

- [***Conservation of Electrical Charge***] In any reaction, the total charge of all the particles entering the reaction must be the same as the total charge of all the particles after the reaction. For example, in the reaction $v_e + d \rightarrow u + e^-$, $0 + (-\frac{1}{3}) = (+\frac{2}{3}) + (-1)$. Similarly, $v_\mu + d \rightarrow u + \mu^-$.

Consider the reverse process by swallowing one of Argon's electrons: ${}^{37}_{18}Ar + e^- \rightarrow {}^{37}_{17}Cl + v_e$, called K capture. The electron that triggers the reaction comes from the lowest energy state of the orbital elecgtrons. The elctron can be swallowed by one of the protons in the nucelus turning it into a neutron with weak force at work. This is a possible formation of Neutron Star.

- [***Conservation of Lepton Generation Number***] In any reaction, the total number of particles from each lepton generation must be the same before and after the reaction.

However, there is an anomaly to this rule. In the Beta radioactivity decay, $n \rightarrow p + e^-$ (i.e. $d \rightarrow u + e^-$). Either the Lepton Generation Rule is violated or we have not yet found out how to apply the rule consistently. Also, people have observed that the following decay events happen in nature: $\mu^- \rightarrow e^- + v_\mu + v_?$ and $\tau^- \rightarrow \mu^- + v_\tau + v_?$. However, how do we distinguish the second neutrino?

Physicists call it internal (intrinsic) properties if it exists in a thing as a natural or permanent quality. The lepton number is a **real** property because it can be demonstrated that lepton number is a conserved quantity in experimental particle reactions and usage of lepton number conservation rule helps us to understand reactions that we could not comprehend or predict otherwise.

	Electron-number L_e	**Muon-number L_μ**	**Tauon-number L_τ**
Electron	1	0	0
Electron Neutrino	1	0	0
Muon	0	1	0
Muon Neutrino	0	1	0
Tauon	0	0	1
Tauon Neutrino	0	0	1

Antimatter

Consider the reaction $v_e + d \rightarrow u + e^-$. By definition all quarks have lepton number 0, hence L_e is conserved ($1 + 0 = 0 + 1$). The total electron number, muon number and tauon number are separately conserved in all reactions.

Consider $\mu^- \rightarrow e^- + v_\mu + v_?$ again:

$$\begin{array}{ccccc} & \mu^- \rightarrow & e^- + & v_\mu + & v_? \\ L_\mu & 1 = & 0 & +\ 1\ + & ? \\ L_e & 0 = & 1 & +\ 0\ + & ? \end{array}$$

We get $L_\mu = 0$ and $L_e = -1$, a new particle. The presence of the mysterious neutrino above is deduced entirely through trying to conserve electron number. We introduce a new type of leptons (anti-particles). We now call the mysterious neutrino with $L_e = -1$ **electron antineutrino**, $\overline{v}_e$. When electron antineutrino reacts with p ($\overline{v}_e + p \rightarrow n + e^+$), we get the **antielectron**.Similarly, we have **muon antineutrino**. When muon antineutrino reacts with p ($\overline{v}_\mu + p \rightarrow n + \mu^+$), we get the **antimuon**.

Similar to the lepton number, we have the **baryon number** for quarks as their internal label. All quarks have baryon number $\frac{1}{3}$ and all leptons have baryon number 0. In all reactions the total baryon number of the particles before the reaction must be the same as the total baryon number after the reaction. The baryon number of all quarks is $+\frac{1}{3}$ and the baryon number of all antiquarks is $-\frac{1}{3}$.

In 1928, Dirac suspected that quantum theory needs to be consistent with special relativity and looked for a Schrodinger equation equivalent. Applying the equation to hydrogen atoms like Bohr, Dirac found an equation $(-i\overrightarrow{\alpha} \cdot \overrightarrow{\nabla} + \beta m)\Psi = E\Psi$. Dirac reduced the equation to $a\Psi^2 + b\Psi + c = 0$ and it seems that Ψ has 2 solutions. Dirac initially interpreted that one solution describes the electron wonderfully but the other is a particle with the same mass but opposite electrical charge. He thought that it may be a proton. In 1933, Anderson found a new particle that seems to turn the wrong way in a magnetic field. Dirac unknowingly predicted the existence of antimatter theoretically.

Antimatter is remarkably rare (unseen) in the universe. Dirac interpreted that a vacancy or a hole in the negative energy sea acts like an electron with a positive charge i.e. a positron. To sum up,

- Dirac's one equation seems to describe 2 particles not one.
- In fact, there are more than 2 particles. There is an infinite sea of particles and energy.
- Spin (up and down) comes out naturally from solving Dirac equation. It means it comes about when marrying quantum mechanics with special relativity!

Matter and Antimatter Collisions

Matter and antimatter will destroy each other if they meet. Particle physicists regularly collide (scatter) small amounts of matter and antimatter together at very high energies. New particles may be produced in such a reaction (collisions, scatterings or interactions). One famous collision is called **annihilation**: $e^+ + e^- \rightarrow ???$. Current LHC is sitting on the old Large Electron-Positron Collide (LEP).

If scattering energies are low, all energies of e^+ and e^- simply scatter off each other and nothing happens. If e^+ and e^- move slowly, then they can go into orbit round each other producing a "Bohr like" atom called **positronium**. This system is unstable. At higher energy, we may get $e^+ + e^- \rightarrow \mu^+ + \mu^-$. At even higher energy, we may get $e^+ + e^- \rightarrow \tau^+ + \tau^-$. At even higher energy still, there will be messy events called jets. They are not clean and nice as the previous lepton reactions but jets. When e^+ and e^- react together, the initial combination of particles is such that the electrical charge, lepton number and baryon number all total to zero. This allows the reaction to produce almost anything, provided the combination adds to zero again. Jets eventually materialize to new particles called **hadrons** (heavy particles, namely baryons and mesons). In fact, as energy thresholds increases, more and more particles are produced (mostly mesons). The 3 jet debris are manifestations of quark, anti-quark and gluon.

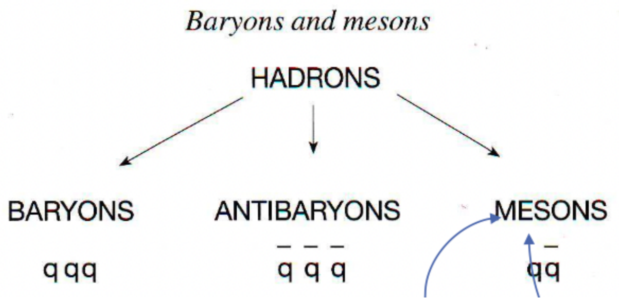
The materilisation of particles and antiparticles is a consequence of $E = mc^2$. So annihilation experiments between matter and antimatter can lead to the production of new matter and antimatter provided they are produced in equal amounts and provided there is enough energy in the reaction, which implies an essential symmetry between them. However, where are all the antimatters in the universe? The creation of the universe would violate the conservation laws unless equal amounts of matter and antimatter are produced.

An application of antimatter is the Positron Emission Tomography (PET) scan, where a positron-emitting fluid flows through the body. Wherever an antimatter positron encounters a normal-matter electron, the pair annihilates and produces particles whose trajectories trace back to the point of annihilation, allowing a computer to reconstruct the shape of structures in the body.

Matter and Matter Collisions

Quarks and gluons are collectively called **partons** (given by Feynmann). Parton-parton collisions also produce jets in LHC. High energy particle detectors are shaped like an onion. We get hadrons from partons from the **hadronization** process. The process is still not understood. There is a theory called Quantum Chromodynamics (QCD).

LHC detectors detect hadrons as final state particles. Baryons (protons and neurons) are part of a larger scheme of particles called hadrons that include mesons. Mesons were first proposed for interaction between nucleons, the intermediate mass between electrons and protons.



Quarks combine to form hadrons in LHC. It is impossible to calculate how much of the mass of a proton is due to the masses of the quarks inside and how much is due to the interaction energies between the quarks, because it is impossible to isolate an individual quark. But we expect that *Down* quark is slightly more massive than the *up* quark.

The masses of the quarks (in GeV/c^2).

Charge	1st generation	2nd generation	3rd generation
+2/3	up 0.33	charm 1.58	top 175[1]
−1/3	down ~0.33	strange 0.47	bottom 4.58

Hadrons

Like leptons, we also have a table for intrinsic properties of hadrons.

The quark flavour numbers.

Quark	U	D	C	S	T	B
up	1	0	0	0	0	0
down	0	−1	0	0	0	0
charm	0	0	1	0	0	0
strange	0	0	0	−1	0	0
top	0	0	0	0	1	0
bottom	0	0	0	0	0	−1

The flavour numbers belong to each quark, not to each generation as in the case of the leptons. There is also no common flavour number to the *up* and *down* quarks. Antiquarks also have flavoured numbers.

The strong force exists only between quarks. HEP theory says that the strong force is flavour independent. The strong force ennsures that quarks and antiquarks can only stick in groups of $3q$ (baryons, qqq or $\overline{q}\,\overline{q}\,\overline{q}$) or $2q$ (mesons, $q\overline{q}$ or $\overline{q}q$).

Quantum Electrodynamics (QED)

QED is a quantum theory for electromagnetism (e^+, e^-) which states that the force is mediated by a particle none other than the usual photon. It is impossible to look directly at the reaction as this would mean interacting with the particles before or during the reaction. For this reason we treat the region in the bubble as a region of ignorance and confine ourselves to trying to calculate the probabilites of different reaction. Once we have calculated a full set of probabilities for all the possibilities, we can work out something about what might be going on inside the bubble.

No one can tell the whole story. All we can say is that the disturbance in the electromagnetic field is very complicated and that each diagram represetns an approximation to the actual physical process. By adding together all the diagrams we can get a mathematically correct answer (similar to principle of superposition). We can also represent antiparticles with a twist in time.

QED theory implies that the gluon force between quarks increases with distance.

Electro-Weak Unification

QED was stunningly successful. It was discovered that the theory of Weak Nuclear force is found to be somewhat similar to QED, so it becomes possible to develop a single mathematical theory in an unified way like Maxwell's E&M. The first unsuccessful version introduces equivalent bosons but cannot explanin why weak force is short range. The successful version introduces massive bosons W^+, W^- and Z^0 (neutral).

So far all exchange particles are in principle massless, but the Higgs field interacting with the bosons gives them very large masses. Additionally, photons appeared. Amazingly it can be emitted with very much less energy called off-mass shell. It is a distrubance in the field that has been forced into existence rather than appearing naturally. Due to the off-mass shell phenomenon, a photon can have mass too. After all, we can never observe them and they are virtual particles.

Vacuum

The photon's coupling to the background electron field has caused an electron-positron pair to materialise out of the background in vaccum. The elctron and positron that materialise and dematerialise again must be off-mass shell. Vacuum physics is much more complicated than one might have thought. It seems that the electron and positron materialise out of the vacuum. The energy required has been donated from the store present in the vacuum and returned again when the 2 particles disappear. The vacuum is a seething mass of such materialisations driven by the store of energy present.

The Casimir effect happens in vacuum. The existence of ground state fluctuations has been confirmed experimentally by the Caimir effect, a slight force between parallel metal plates. The number of wavelengths that can fit between the plates reduces. The energy density of ground state fluctuations between the plates is less than the density outside, causing the plates to draw together. The energy density of ground state fluctuations is greater outside the plates.

Susy

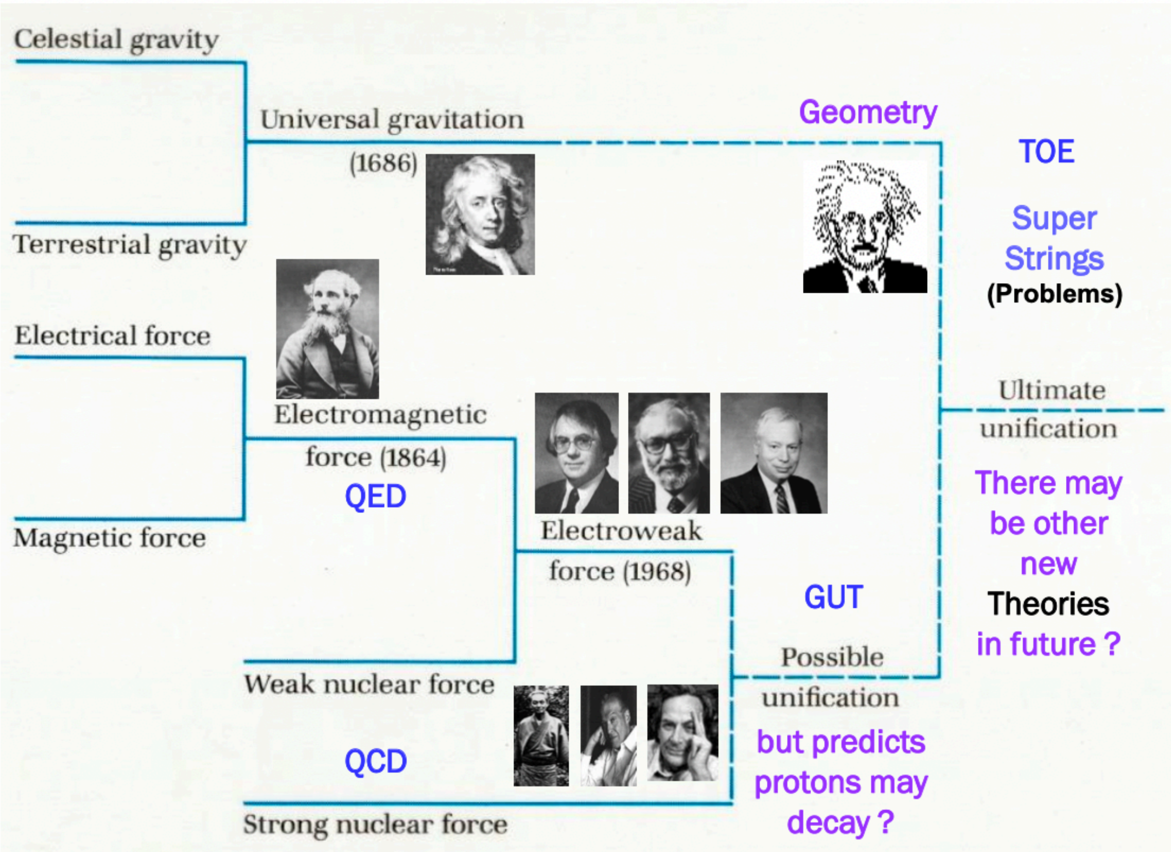
Modern theory tries to link fermions and bosons in a unified scheme. To do this unification, we need to allow fermions to convert to bosons and vice versa. The price to pay is to introduce more particles. The theory implies that there must be a fermion equivalent to every boson and vice versa, hence the Supersymmetry theory.

Susy Theory (Supersymmetry)

Ordinary Matter & Interactions (Fermions & Bosons)	Super matter partner Introduce more Matter
Quarks	Squarks
Leptons	Sleptons
W Particles	Winos, Zinos
Higgs particle	Higgsino
Gluon	Gluino
Photon	Photino
Graviton	Gravitino

String Theory

Susy marries strings: super strings. Each mode of vibration gives the superstring the properties of a fundamental particle. By allowing the strings to interact with each other all the fundamental forces, including gravity can be incorporated into one unified force or scheme. The price is extra dimensions. Time of interaction becomes uncertain.



If LHC discovers supersymmetric top quarks, how would you propose to name them? STOP!

The End

Nature as we know it is probabilistic. Heisenberg commented on Einstein that Einstein was unable to accept Planck's quantum theory since it did not correspond to his philosophical coneptions of the task of the exact sciences. He felt it disturbing that natual laws should have to related to the probability of objective processes.

Important points to note:

- When light is reflected by 90° (as in the Mach-Zehnder Interferometer), it undergoes a $\frac{\pi}{2}$ phase shift.
- When a beam of light is split into two by a 50/50 beam splitter, we add a normalisation factor of $\frac{1}{\sqrt{2}}$.

E.g. When the light in state $|1\rangle$ is split by the beam splitter, we can represent it as such:

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}|2\rangle + i\frac{1}{\sqrt{2}}|3\rangle \tag{1}$$

4) The Periodic table has about 92 naturally occurring elements and has been extended every now and then by artificial means ~ 105. By considering the Bohr type atom (with electron moving around orbits), could you discuss the possibility of making elements with 888 atomic number? 888 seem to be auspicious Chinese number!

Remember that the Bohr model of the atom pictures electrons 'orbiting' around the centre nucleus. The centripetal acceleration from this circular motion comes from the Coulomb force, or the electromagnetic attraction between the negatively charged electrons and the positively charged nucleus.

$$\begin{aligned} F &= \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \\ &= \frac{mv^2}{r} \end{aligned} \tag{6}$$

The Bohr's quantisation condition tells us that the angular momentum of an electron in an 'orbit' around an atom has to be an integral multiple of the reduced Planck's constant.

$$\begin{aligned} L &= n\hbar \\ &= mrv \end{aligned} \tag{7}$$

In this case, we are interested in the innermost 'orbit', where $n = 1$. Plugging in the relevant values, and using $Z = 888$, we find that

$$\begin{aligned} v &= 1.941 * 10^9 \text{ms}^{-1} \\ &= 6.48\, c \end{aligned} \tag{8}$$

Question: Is this allowed by relativity?

Table 11.4.2: Baryon Quarks

Name	**Symbol**	**Quarks**	**Charge (unit of e)**	**Spin (s)**	**Mass** (GeV/c^2)
Proton	p	**u u d**	1	1/2	0.938
Neutron	n	**u d d**	0	1/2	0.940
Delta plus plus	Δ^{++}	**u u u**	2	3/2	1.232
Delta plus	Δ^{+}	**u u d**	1	3/2	1.232
Delta zero	Δ^{0}	**u d d**	0	3/2	1.232
Delta minus	Δ^{-}	d d d	-1	3/2	1.232
Lambda zero	Λ^{0}	u d s	0	1/2	1.116
Positive sigma	Σ^{+}	u u s	1	1/2	1.189
Neutral sigma	Σ^{0}	u d s	0	1/2	1.192
Negative xi	Ξ^{-}	s d s	-1	1/2	1.321
Neutral xi	Ξ_{0}	s u s	0	1/2	1.315
Omega minus	Ω^{-}	s s s	-1	3/2	1.672
Charmed lambda	Λ_{C+}	u d c	1	1/2	2.281
Charmed bottom	Λ_{b0}	u d b	0	1/2	5.612

Table 11.4.3: Meson Quarks

Name	Symbol	Quarks	Charge (**e**)	Spin	Mass (GeV/c^2)
Positive pion	π^{+}	$u\bar{d}$	1	0	0.140
Positive rho	ρ^{+}	$u\bar{d}$	1	1	0.768
Negative pion	π^{-}	$\bar{u}d$	-1	0	0.140
Negative rho	ρ^{-}	$\bar{u}d$	-1	1	0.768
Neutral Pion	π^{0}	$\bar{u}u$ or $\bar{d}d$	0	0	0.135
Neutral eta	η^{0}	$\bar{u}u$, $\bar{d}d$ or $\bar{s}s$	0	0	0.547
Positive kaon	K^{+}	$u\bar{s}$	1	0	0.494
Neutral kaon	K^{0}	$d\bar{s}$	0	0	0.498
Negative kaon	K^{-}	$\bar{u}s$	-1	0	0.494
J/Psi	J/ψ	$\bar{c}c$	0	1	3.10
Charmed eta	η_{0}	$c\bar{c}$	0	0	2.98
Neutral D	D^{0}	$\bar{u}c$	0	0	1.86
Neutral D	D^{*0}	$\bar{u}c$	0	1	2.01
Positive D	D^{+}	$\bar{d}c$	1	0	1.87
Neutral B	B^{0}	$\bar{d}b$	0	0	5.26
Upsilon	Υ	$b\bar{b}$	0	1	9.46