ST3131 Regression Analysis

AY2021/22 Semester 2

Multiple Linear Regression Model

- Simple Linear Regression Model

  $$Y = \beta_0 + \beta_1 X + \epsilon$$

  - Fitted Regression Function:

    $$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

  - ANOVA Table:

Source	d.f.	SS	MS	F
Regression	1	SSR	MSR	MSR/MSE
Error	$n-2$	SSE	MSE	
Total	$n-1$	SST		

  - Coefficient of Determination: It explains how much of the variation in Y can be explained by X.

    $$R^2 = \frac{SSR}{SST}$$

    - Adjusted R^2: $R_a^2 = \frac{n-1}{n-p-1}R^2 - \frac{p}{n-p-1}$, where p is the number of predictors and equals 1 for simple LRM.

  - Significance Test
    - Significance F-Test
      1. $H_0: \beta_1 = 0$ (no linear relationship)
      2. $H_1: \beta_1 \neq 0$
      3. Test statistics: $F = \frac{MSR}{MSE}$
      4. Under H_0, $F \sim F_{1,n-2}$.
      5. If $F > F_{1,n-2}(\alpha)$, H_0 is rejected at level α.
    - Two-sided Test for β_1
      1. $H_0: \beta_1 = 0$
      2. $H_1: \beta_1 \neq 0$
      3. Test statistics: $T_{0_{\beta_1}} = \frac{\hat{\beta}_1}{s(\hat{\beta}_1)}$
      4. Under H_0, $T_{0_{\beta_1}} \sim t_{n-2}$.
      5. If $\left|T_{0_{\beta_1}}\right| > t_{n-2}\left(\frac{\alpha}{2}\right)$, reject H_0.
      6. p-value: $p = P\left(|t_{n-2}| > T_{0_{\beta_1}}\right)$. If $p < \alpha$, reject H_0.
    - One-sided Test for β_1
      1. $H_0: \beta_1 \leq 0$
      2. $H_1: \beta_1 > 0$
      3. Decision rule: $T_{0_{\beta_1}} > t_{n-2}(\alpha)$
    - Confidence Interval of β_1:

      $$\left[\hat{\beta}_1 - t_{n-2}\left(\frac{\alpha}{2}\right)s(\hat{\beta}_1), \hat{\beta}_1 + t_{n-2}\left(\frac{\alpha}{2}\right)s(\hat{\beta}_1)\right]$$

    - Significance test for β_0 is similar.

- R Codes

Input data into a dataframe
```
q1.dat=read.table("~/Downloads/",header=TRUE)
```

Make the variables available
```
y=q1.dat$Y  (replace Y with the header name)
x=q1.dat$X
```

Plot a graph of Y against X
```
plot(x,y,xlabel="x",ylabel="y")
```

Fit a simple linear regression model to the data
```
q1.fit=lm(y~x)
summary(q1.fit)
```

Plot the fitted line in blue
```
abline(q1.fit,col="blue")
```

- Multiple Linear Regression Model

  $$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \epsilon$$

  - Observations: $y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_p x_{ip} + \epsilon_i$, $i = 1,2,\ldots,n$ where e_i's are i.i.d. $\sim N(0,\sigma^2)$.
  - MLE of $\boldsymbol{\beta}$: Same as LSE, $\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}$
  - LSE of σ^2: $\hat{\sigma}^2 = \frac{1}{n-p-1}\left\|\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}\right\|^2$
  - MLE of σ^2: $\hat{\sigma}^2 = \frac{1}{n}\left\|\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}}\right\|^2 = \frac{n-p-1}{n}\hat{\sigma}^2_{LSE}$
  - ANOVA Table:

Source of Variation	SS	df	MS	F-statistic
Regression	SSR	p	$MSR = \frac{SSR}{p}$	MSR/MSE
Error	SSE	$n-p-1$	$MSE = \frac{SSE}{n-p-1}$	
Total	SST	$n-1$		

  - Coefficient of Determination:

    $$R^2 = \frac{\sum_{i=1}^n(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n(y_i - \bar{y})^2} = \frac{SSR}{SST} = corr(\boldsymbol{y}, \hat{\boldsymbol{y}})^2$$

    - Adjusted R^2: $R_a^2 = R^2 - \frac{p}{n-p-1}(1 - R^2)$

  - Significance Test
    - Significance F-Test: Is there a linear relationship?
      1. $H_0: \beta_1 = \cdots = \beta_p = 0$
      2. $H_1: \neg H_0$
      3. Test statistics: $F = \frac{MSR}{MSE}$
      4. Under H_0, $F \sim F_{p,n-p-1}$.
      5. If $F > F_{p,n-p-1}(\alpha)$, reject H_0.
    - Wald Statistics

      $$W = \hat{\boldsymbol{\beta}}^T\hat{\boldsymbol{V}}^{-1}\hat{\boldsymbol{\beta}}$$

      In Multiple LRM, $F = \frac{W}{p}$, where p is number of predictors (dimension of $\hat{\boldsymbol{\beta}}$).

      Simply divide by dimension of β or V

    - Two-sided Individual t-test for β_j
      1. $H_0: \beta_j = 0$
      2. $H_1: \beta_j \neq 0$
      3. Test statistics: $t = \frac{\hat{\beta}_j}{\hat{\sigma}_j}$
      4. Under H_0, $t \sim t_{n-p-1}$.
      5. If $|t| \geq t_{n-p-1}\left(\frac{\alpha}{2}\right)$, reject H_0.
    - General t-test:
      1. $H_0: \sum_{j=0}^p \boldsymbol{c}_j\beta_j = d$
      2. $H_1: \neg H_0$
      3. Test statistics: $t = \frac{\boldsymbol{c}^T\hat{\boldsymbol{\beta}} - d}{\sqrt{\boldsymbol{c}^T\hat{\boldsymbol{V}}\boldsymbol{c}}}$
      4. Under H_0, $t \sim t_{n-p-1}$.
      5. If $|t| \geq t_{n-p-1}\left(\frac{\alpha}{2}\right)$, reject H_0.
    - Confidence Interval for β_j:

      $$\left[\hat{\beta}_j - \hat{\sigma}_j t_{n-p-1}\left(\frac{\alpha}{2}\right), \hat{\beta}_j + \hat{\sigma}_j t_{n-p-1}\left(\frac{\alpha}{2}\right)\right]$$

- Confidence Interval for $\boldsymbol{\beta}$:

$$\left[\boldsymbol{c}^T\widehat{\boldsymbol{\beta}} - \sqrt{\boldsymbol{c}^T\widehat{\boldsymbol{V}}\boldsymbol{c}} \times t_{n-p-1}\left(\frac{\alpha}{2}\right), \boldsymbol{c}^T\widehat{\boldsymbol{\beta}} + \sqrt{\boldsymbol{c}^T\widehat{\boldsymbol{V}}\boldsymbol{c}} \times t_{n-p-1}\left(\frac{\alpha}{2}\right)\right]$$

- $100(1-\alpha)\%$ Confidence Upper Bound for β_j:

$$\beta_j \le \hat{\beta}_j + \hat{\sigma}_j t_{n-p-1}(\alpha)$$

- $100(1-\alpha)\%$ Confidence Lower Bound for β_j:

$$\beta_j \ge \hat{\beta}_j - \hat{\sigma}_j t_{n-p-1}(\alpha)$$

- R Codes

Fit a multiple linear regression model to the data

```
q1.fit=lm(y~x1+x2)
summary(q1.fit)
```

Isolate an expression in the model

```
I(…)
```

Extract fitted values, residues, estimated coefficients and covariance matrix

```
yhat=q1.fit$fitted
r=q1.fit$resid
b=q1.fit$coef
V=vcov(q1.fit)
```

Compute squared Pearson's correlation coefficient between $\boldsymbol{y}$ *and* $\hat{\boldsymbol{y}}$

```
cor(y,yhat)^2
```

p-value of $P(t_{df} < x)$ *and* $P(t_{df} > x)$

```
pt(x,df)
1-pt(x,df)
```

Linear Regression Model for Analysis of Variance

- Dummy Variables: If a factor has a levels, it can be represented by $a-1$ dummy variables.

$$u_k = \begin{cases} 1, \text{if level k} \\ 0, \text{otherwise} \end{cases}$$

  - Example: Considering the difference between region 2 and 4, region 1 and 3 and region 1 and 4, then $c = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

- ANOVA Table:

Sourse	d.f.	SS	MS	F-test
A	a-1	SSA	MSA	MSA/MSE
B	b-1	SSB	MSB	MSB/MSE
AB	(a-1)(b-1)	SSAB	MSAB	MSAB/MSE
Error	n-ab	SSE	MSE	–
Total	n-1	SST	–	–

  - n_{ERR} here is for interaction model. For main effect model, add $(a-1)(b-1)$.

- Main Effect Model (2 factors)

$$y_i = \beta_0 + \sum_{k=2}^{a} \alpha_k u_{ik} + \sum_{j=2}^{b} \beta_j v_{ij} + \epsilon_i$$

  - Adjusted Covariance Matrix:

$$\widehat{\boldsymbol{V}} = \frac{\sigma_I^2}{\sigma_M^2}\widetilde{\boldsymbol{V}}$$

  - Adjusted F-Statistics:

$$F_a = \frac{\sigma_M^2}{\sigma_I^2} \cdot F$$

  - Adjusted t-Statistics:

$$t_a = \frac{\sigma_M^2}{\sigma_I^2} \cdot t$$

  - When the cells are **imbalanced**, the F-statistics in the ANOVA table are not all correct.

- Interaction Model (2 factors)

$$y_i = \beta_0 + \sum_{k=2}^{a} \alpha_k u_{ik} + \sum_{j=2}^{b} \beta_j v_{ij} + \sum_{k=2}^{a}\sum_{j=2}^{b} \gamma_{kj} u_{ik} v_{ij} + \epsilon_i$$

  - Significance Test

Wald Statistics: When dimension is 1, $\boldsymbol{W = \frac{\gamma^2}{V}}$

    - Overall Interaction Effect:
      1. $H_0: \gamma_{kj} = 0$ for all k, j
      2. $H_1: \neg H_0$
      3. Wald Statistics: $W = \widehat{\boldsymbol{\gamma}}^T \widehat{\boldsymbol{V}}_\gamma^{-1} \widehat{\boldsymbol{\gamma}}$
      4. F-Statistics: $F = \frac{W}{(a-1)(b-1)}$
      5. Under H_0, $F \sim F_{(a-1)(b-1), n-ab}$.
      6. If $F > F_{(a-1)(b-1), n-ab}(\alpha)$, reject H_0.

    - Particular Interaction Effect
      1. $H_0: \gamma_{kj} = 0$
      2. $H_1: \gamma_{kj} \neq 0$
      3. Test statistics: $T_c = \frac{c^T\gamma}{\sqrt{c^T V_\gamma c}}$. The choice of $\boldsymbol{c}$ and $\boldsymbol{\gamma}$ depends on the hypothesis (*L3S38*).
      4. Under H_0, $T_c \sim t_{n-ab}$.

- R Codes

Fit an interaction model of y, x1, x2

```
int=lm(y~x1*x2,data=q2.dat)
summary(int)
anova(int)
```

Fit a main effect model of y, x1, x2 where x1 is ***before*** *x2*

```
main=lm(y~x1+x2,data=q2.dat)
summary(main)
anova(main)
```

Adjust the statistics in main effect model

```
sigma.m=summary(main)$sigma
sigma.i=summary(int)$sigma
b=main$coef
V=vcov(main)
V.a=V*sigma.i^2/sigma.m^2
round(V.a,4)  (may not be necessary)
sd.b=sqrt(diag(V.a))
t=b/sd.b
p=1-pt(abs(t),28)  (may need to adjust, note that the degree of freedom follows that of interaction model)
```

Calculate Wald Statistics and F-Statistics

```
b=int$coeff
V=vcov(int)
b.x=b[6:9]  (suppose only these entries are relevant, note that
             b[0] and V[0][0] corresponds to the intercept)
V.x=V[6:9][6:9]
W=t(b.x)%*%solve(v.x)%*%b.x
F=W/4  (suppose (a − 1)(b − 1) = 4)
```

Multiple Comparisons

- Scheffe's Criterion: $c_\alpha = \sqrt{(a-1)F_{a-1,n_{ERR}}(\alpha)}$, where a is the number of groups, n_{ERR} = number of groups × sample size per group – a.
  - Definition: Let $\mathcal{C}$ denote the set of all contrasts, then $P\left(\max_{c\in\mathcal{C}}|T_c| \geq c_\alpha\right) = \alpha$.
  - A contrast is significant if its test statistics exceeds c_α.

- Studentized Range Statistics:
$$q_{a,n_{ERR}} = \sqrt{n}(\max \bar{Y}_i - \min \bar{Y}_i)/\hat{\sigma}$$
  - Q-Statistics:
$$Q_{ij} = \begin{cases} \dfrac{\sqrt{n}|\bar{Y}_i - \bar{Y}_j|}{\hat{\sigma}} & \text{if } n_1 = \cdots = n_a = n \\ \dfrac{\sqrt{\dfrac{2n_in_j}{n_i+n_j}}|\bar{Y}_i - \bar{Y}_j|}{\hat{\sigma}} & \text{otherwise} \end{cases}$$
    - $|T_{ij}| = Q_{ij}/\sqrt{2}$
  - T-Statistics:
$$T_{ij} = \begin{cases} \dfrac{\hat{\beta}_i}{s.d(\hat{\beta}_i)} & \text{if } j = 1 \\ \dfrac{\hat{\beta}_i - \hat{\beta}_j}{[Var(\hat{\beta}_i) + Var(\hat{\beta}_j) - 2Cov(\hat{\beta}_i, \hat{\beta}_j)]^{\frac{1}{2}}} & \text{otherwise} \end{cases}$$
Read from model summary (if $j = 1$); **Use adjusted matrix** (otherwise)
    - T-Statistics needs to be compared with $\frac{q_{a,n_{ERR}}(\alpha)}{\sqrt{2}}$.

- Tukey's Criterion: $\frac{q_{a,n_{ERR},\alpha}}{\sqrt{2}}$.
  - Definition: Let $\mathcal{P}$ denote the set of all pairwise comparison, then $P\left(\max_{c\in\mathcal{C}}|T_c| \geq q_\alpha\right) \geq P\left(\max_{c\in\mathcal{P}}|T_c| \geq q_\alpha\right) = \alpha$.

- Bonferroni's Criterion: $t_{n_{ERR}}\left(\frac{\alpha}{k}\right)$, where k is the number of prespecified contrasts. If two-sided, take $t_{n_{ERR}}\left(\frac{\alpha}{2k}\right)$.

- How to choose from the three criteria?
  - If it is only to control overall type I error rate, all three criteria can be used.
  - Bonferroni can be used if the given contrasts are the only concern.
  - Tukey can be used if the given contrasts are part of pairwise contrasts.
  - In general, Tukey is smaller than Scheffe. In consideration of efficiency, Scheffe is excluded.
  - Between Tukey and Bonferroni, the one with smaller value should be chosen.

- R Codes

Read Categorical Data
```
z=factor(region)
```
(region has 4 categories)

Fit a model of y, x1, x2, x3 with all the main effect terms and two-variable interaction terms
```
fit=lm(y~x1+x2+x3+x1:x2+x1:x3+x2:x3,data=q3.dat)
summary(fit)
```

Variable Selection

- Akaike's Information Criterion: $AIC = -2\log L(\hat{\beta}_M, \hat{\sigma}_M^2) + 2j_M$, where $\hat{\beta}_M, \hat{\sigma}_M$ are **MLE** under model M, j_M is the number of predictors in M.
  - Under Multiple LRM, $AIC = n\log(\hat{\sigma}_M^2) + 2j_M + C$, where $C = n(\ln 2\pi + 1)$.

- Bayesian Information Criterion: $BIC = -2\log L(\hat{\beta}_M, \hat{\sigma}_M^2) + j_M \ln n$.
  - Under Multiple LRM, $BIC = n\log(\hat{\sigma}_M^2) + C + j_M \ln n$, where $C = n(\ln 2\pi + 1)$.

- Cross-Validation (CV) Scores:
  - Leave-out-one CV Score:
$$CV = \frac{1}{n}\sum_{i=1}^{n}[y_i - \boldsymbol{x}_i^T\hat{\boldsymbol{\beta}}^{-i}]^2$$
    - For the model $Y = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \epsilon$, and n observations (y_1, x_{11}, x_{12}) …, the Leave-out-one CV:
$$CV = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{\beta}_0^{-i} - \hat{\beta}_1^{-i}x_{i1} - \hat{\beta}_2^{-i}x_{i2})^2$$
  - d-fold CV Score:
$$CV = \frac{1}{d}\sum_{j=1}^{d}\|\boldsymbol{y}_j - \boldsymbol{X}_j\hat{\boldsymbol{\beta}}^{-j}\|^2$$

- Sequential Procedures
  - Forward Selection
  - Backward Selection
  - Upwards Stepwise Selection
  - Downwards Stepwise Selection

- Penalised Likelihood Approach
  - Least Absolute Shrinkage and Selection Operator (LASSO)

- R Codes

Calculate AIC, BIC, CV
```
library(boot)
M=glm(y~x1+x2+x3+x4+x5+x6,data=q4.dat)
AIC(M)
BIC(M)
cv.glm(q4.dat,M)$delta[2]
```

Carry out different selection procedures
```
library(MASS)
lower.m=lm(y~1,data=q4.dat)
upper.m=lm(y~x1+x2+x3+x4+x5+x6+x7,data=q4.dat)

# forward selection
forward=stepAIC(lower.m,scope=list(lower=~1,upper=~x1+x2+x3+x4+x5+x6+x7),direction="forward")
summary(forward)

# backward selection
backward=stepAIC(upper.m,scope=list(upper=~x1+x2+x3+x4+x5+x6+x7,lower=~1),direction="backward")
summary(backward)

# stepwise upward selection
stepUp=stepAIC(lower.m,scope=list(lower=~1,upper=~x1+x2+x3+x4+x5+x6+x7),direction="both")
summary(stepUp)

# stepwise downward selection
stepDown=stepAIC(upper.m,scope=list(lower=~1,upper=~x1+x2+x3+x4+x5+x6+x7),direction="both")
summary(stepDown)
```

Model Diagnostics

- Checking Non-linearity
  - Plot of Pearson's residual against fitted values
  - Plot of Pearson's residual against predictors
  - Scatter plot of response against predictors
  - If any non-linear trend shows up, it is an indication of non-linearity.

- Checking Homogenity (constant variance)
  - Plot of Pearson's residual against fitted values
  - Plot of Pearson's residual against predictors
  - If variances are not constant, the vertical range of the residues will have obvious changes along the x-axis.

- Checking Independence
  - Plot of Pearson's residual against time/space where the observations are obtained

- Checking Normality
  - Distribution plot of Pearson's residual, such as box plot, histogram, etc.
  - Normal probability plot of residual (Q-Q plot)
  - If normality holds, the points of normal probability plot should roughly fall along a straight line.

- Checking Missing Predictors
  - Plot of Residuals against other predictors not included in the model

- R Codes

Prepare raw materials

```
yhat=model.fit$fitted.values
r=residuals(model.fit,type="pearson")
h=hatvalues(model.fit,type="diagonal")
infl=influence(model.fit,do.coef=FALSE)
rsta=rstandard(model.fit,infl,type="pearson")
rstu=rstudent(model.fit,infl,type="pearson")
cook=cooks.distance(model.fit,infl,res=infl$pear
.res,dispersion=summary(model.fit)$dispersion,ha
t=infl$hat)
```

Fit a model of y, x1, x2, x3 with all the main effect terms and two-variable interaction terms

```
fit=lm(y~x1+x2+x3+x1:x2+x1:x3+x2:x3,data=q3.dat)
summary(fit)
```

t-distribution table

Areas in the upper tail are given along the top of the table. Critical t* values are given in the table.

df	0.1	0.05	0.025	0.02	0.01	0.005
1	3.078	6.314	12.706	15.895	31.821	63.657
2	1.886	2.920	4.303	4.849	6.965	9.925
3	1.638	2.353	3.182	3.482	4.541	5.841
4	1.533	2.132	2.776	2.999	3.747	4.604
5	1.476	2.015	2.571	2.757	3.365	4.032
6	1.440	1.943	2.447	2.612	3.143	3.707
7	1.415	1.895	2.365	2.517	2.998	3.499
8	1.397	1.860	2.306	2.449	2.896	3.355
9	1.383	1.833	2.262	2.398	2.821	3.250
10	1.372	1.812	2.228	2.359	2.764	3.169
11	1.363	1.796	2.201	2.328	2.718	3.106
12	1.356	1.782	2.179	2.303	2.681	3.055
13	1.350	1.771	2.160	2.282	2.650	3.012
14	1.345	1.761	2.145	2.264	2.624	2.977
15	1.341	1.753	2.131	2.249	2.602	2.947
16	1.337	1.746	2.120	2.235	2.583	2.921
17	1.333	1.740	2.110	2.224	2.567	2.898
18	1.330	1.734	2.101	2.214	2.552	2.878
19	1.328	1.729	2.093	2.205	2.539	2.861
20	1.325	1.725	2.086	2.197	2.528	2.845
21	1.323	1.721	2.080	2.189	2.518	2.831
22	1.321	1.717	2.074	2.183	2.508	2.819
23	1.319	1.714	2.069	2.177	2.500	2.807
24	1.318	1.711	2.064	2.172	2.492	2.797
25	1.316	1.708	2.060	2.167	2.485	2.787
26	1.315	1.706	2.056	2.162	2.479	2.779
27	1.314	1.703	2.052	2.158	2.473	2.771
28	1.313	1.701	2.048	2.154	2.467	2.763
29	1.311	1.699	2.045	2.150	2.462	2.756
30	1.310	1.697	2.042	2.147	2.457	2.750
31	1.309	1.696	2.040	2.144	2.453	2.744
32	1.309	1.694	2.037	2.141	2.449	2.738
33	1.308	1.692	2.035	2.138	2.445	2.733
34	1.307	1.691	2.032	2.136	2.441	2.728
35	1.306	1.690	2.030	2.133	2.438	2.724
36	1.306	1.688	2.028	2.131	2.434	2.719
37	1.305	1.687	2.026	2.129	2.431	2.715
38	1.304	1.686	2.024	2.127	2.429	2.712
39	1.304	1.685	2.023	2.125	2.426	2.708
40	1.303	1.684	2.021	2.123	2.423	2.704
41	1.303	1.683	2.020	2.121	2.421	2.701
42	1.302	1.682	2.018	2.120	2.418	2.698
43	1.302	1.681	2.017	2.118	2.416	2.695
44	1.301	1.680	2.015	2.116	2.414	2.692
45	1.301	1.679	2.014	2.115	2.412	2.690
46	1.300	1.679	2.013	2.114	2.410	2.687
47	1.300	1.678	2.012	2.112	2.408	2.685
48	1.299	1.677	2.011	2.111	2.407	2.682
49	1.299	1.677	2.010	2.110	2.405	2.680
50	1.299	1.676	2.009	2.109	2.403	2.678

df	0.1	0.05	0.025	0.02	0.01	0.005
51	1.298	1.675	2.008	2.108	2.402	2.676
52	1.298	1.675	2.007	2.107	2.400	2.674
53	1.298	1.674	2.006	2.106	2.399	2.672
54	1.297	1.674	2.005	2.105	2.397	2.670
55	1.297	1.673	2.004	2.104	2.396	2.668
56	1.297	1.673	2.003	2.103	2.395	2.667
57	1.297	1.672	2.002	2.102	2.394	2.665
58	1.296	1.672	2.002	2.101	2.392	2.663
59	1.296	1.671	2.001	2.100	2.391	2.662
60	1.296	1.671	2.000	2.099	2.390	2.660
61	1.296	1.670	2.000	2.099	2.389	2.659
62	1.295	1.670	1.999	2.098	2.388	2.657
63	1.295	1.669	1.998	2.097	2.387	2.656
64	1.295	1.669	1.998	2.096	2.386	2.655
65	1.295	1.669	1.997	2.096	2.385	2.654
66	1.295	1.668	1.997	2.095	2.384	2.652
67	1.294	1.668	1.996	2.095	2.383	2.651
68	1.294	1.668	1.995	2.094	2.382	2.650
69	1.294	1.667	1.995	2.093	2.382	2.649
70	1.294	1.667	1.994	2.093	2.381	2.648
71	1.294	1.667	1.994	2.092	2.380	2.647
72	1.293	1.666	1.993	2.092	2.379	2.646
73	1.293	1.666	1.993	2.091	2.379	2.645
74	1.293	1.666	1.993	2.091	2.378	2.644
75	1.293	1.665	1.992	2.090	2.377	2.643
76	1.293	1.665	1.992	2.090	2.376	2.642
77	1.293	1.665	1.991	2.089	2.376	2.641
78	1.292	1.665	1.991	2.089	2.375	2.640
79	1.292	1.664	1.990	2.088	2.374	2.640
80	1.292	1.664	1.990	2.088	2.374	2.639
81	1.292	1.664	1.990	2.087	2.373	2.638
82	1.292	1.664	1.989	2.087	2.373	2.637
83	1.292	1.663	1.989	2.087	2.372	2.636
84	1.292	1.663	1.989	2.086	2.372	2.636
85	1.292	1.663	1.988	2.086	2.371	2.635
86	1.291	1.663	1.988	2.085	2.370	2.634
87	1.291	1.663	1.988	2.085	2.370	2.634
88	1.291	1.662	1.987	2.085	2.369	2.633
89	1.291	1.662	1.987	2.084	2.369	2.632
90	1.291	1.662	1.987	2.084	2.368	2.632
91	1.291	1.662	1.986	2.084	2.368	2.631
92	1.291	1.662	1.986	2.083	2.368	2.630
93	1.291	1.661	1.986	2.083	2.367	2.630
94	1.291	1.661	1.986	2.083	2.367	2.629
95	1.291	1.661	1.985	2.082	2.366	2.629
96	1.290	1.661	1.985	2.082	2.366	2.628
97	1.290	1.661	1.985	2.082	2.365	2.627
98	1.290	1.661	1.984	2.081	2.365	2.627
99	1.290	1.660	1.984	2.081	2.365	2.626
100	1.290	1.660	1.984	2.081	2.364	2.626

Table of critical values for the F distribution (for use with ANOVA):

How to use this table:

There are two tables here. The first one gives critical values of F at the p = 0.05 level of significance. The second table gives critical values of F at the p = 0.01 level of significance.

1. Obtain your F-ratio. This has (x,y) degrees of freedom associated with it.

2. Go along x columns, and down y rows. The point of intersection is your critical F-ratio.

3. If your obtained value of F is equal to or larger than this critical F-value, then your result is significant at that level of probability.

An example: I obtain an F ratio of 3.96 with (2, 24) degrees of freedom.

I go along 2 columns and down 24 rows. The critical value of F is 3.40. My obtained F-ratio is larger than this, and so I conclude that my obtained F-ratio is likely to occur by chance with a p<.05.

Critical values of F for the 0.05 significance level:

	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.97	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.33	3.47	3.07	2.84	2.69	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.26
25	4.24	3.39	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.17
31	4.16	3.31	2.91	2.68	2.52	2.41	2.32	2.26	2.20	2.15
32	4.15	3.30	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14
33	4.14	3.29	2.89	2.66	2.50	2.39	2.30	2.24	2.18	2.13
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11

36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15	2.11
37	4.11	3.25	2.86	2.63	2.47	2.36	2.27	2.20	2.15	2.10
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09
39	4.09	3.24	2.85	2.61	2.46	2.34	2.26	2.19	2.13	2.08
40	4.09	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
41	4.08	3.23	2.83	2.60	2.44	2.33	2.24	2.17	2.12	2.07
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.07
43	4.07	3.21	2.82	2.59	2.43	2.32	2.23	2.16	2.11	2.06
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.15	2.09	2.04
47	4.05	3.20	2.80	2.57	2.41	2.30	2.21	2.14	2.09	2.04
48	4.04	3.19	2.80	2.57	2.41	2.30	2.21	2.14	2.08	2.04
49	4.04	3.19	2.79	2.56	2.40	2.29	2.20	2.13	2.08	2.03
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
51	4.03	3.18	2.79	2.55	2.40	2.28	2.20	2.13	2.07	2.02
52	4.03	3.18	2.78	2.55	2.39	2.28	2.19	2.12	2.07	2.02
53	4.02	3.17	2.78	2.55	2.39	2.28	2.19	2.12	2.06	2.02
54	4.02	3.17	2.78	2.54	2.39	2.27	2.19	2.12	2.06	2.01
55	4.02	3.17	2.77	2.54	2.38	2.27	2.18	2.11	2.06	2.01
56	4.01	3.16	2.77	2.54	2.38	2.27	2.18	2.11	2.05	2.01
57	4.01	3.16	2.77	2.53	2.38	2.26	2.18	2.11	2.05	2.00
58	4.01	3.16	2.76	2.53	2.37	2.26	2.17	2.10	2.05	2.00
59	4.00	3.15	2.76	2.53	2.37	2.26	2.17	2.10	2.04	2.00
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
61	4.00	3.15	2.76	2.52	2.37	2.25	2.16	2.09	2.04	1.99
62	4.00	3.15	2.75	2.52	2.36	2.25	2.16	2.09	2.04	1.99
63	3.99	3.14	2.75	2.52	2.36	2.25	2.16	2.09	2.03	1.99
64	3.99	3.14	2.75	2.52	2.36	2.24	2.16	2.09	2.03	1.98
65	3.99	3.14	2.75	2.51	2.36	2.24	2.15	2.08	2.03	1.98
66	3.99	3.14	2.74	2.51	2.35	2.24	2.15	2.08	2.03	1.98
67	3.98	3.13	2.74	2.51	2.35	2.24	2.15	2.08	2.02	1.98
68	3.98	3.13	2.74	2.51	2.35	2.24	2.15	2.08	2.02	1.97
69	3.98	3.13	2.74	2.51	2.35	2.23	2.15	2.08	2.02	1.97
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97
71	3.98	3.13	2.73	2.50	2.34	2.23	2.14	2.07	2.02	1.97
72	3.97	3.12	2.73	2.50	2.34	2.23	2.14	2.07	2.01	1.97
73	3.97	3.12	2.73	2.50	2.34	2.23	2.14	2.07	2.01	1.96
74	3.97	3.12	2.73	2.50	2.34	2.22	2.14	2.07	2.01	1.96
75	3.97	3.12	2.73	2.49	2.34	2.22	2.13	2.06	2.01	1.96
76	3.97	3.12	2.73	2.49	2.34	2.22	2.13	2.06	2.01	1.96
77	3.97	3.12	2.72	2.49	2.33	2.22	2.13	2.06	2.00	1.96
78	3.96	3.11	2.72	2.49	2.33	2.22	2.13	2.06	2.00	1.95
79	3.96	3.11	2.72	2.49	2.33	2.22	2.13	2.06	2.00	1.95
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95
81	3.96	3.11	2.72	2.48	2.33	2.21	2.13	2.06	2.00	1.95
82	3.96	3.11	2.72	2.48	2.33	2.21	2.12	2.05	2.00	1.95
83	3.96	3.11	2.72	2.48	2.32	2.21	2.12	2.05	2.00	1.95
84	3.96	3.11	2.71	2.48	2.32	2.21	2.12	2.05	1.99	1.95
85	3.95	3.10	2.71	2.48	2.32	2.21	2.12	2.05	1.99	1.94

86	3.95	3.10	2.71	2.48	2.32	2.21	2.12	2.05	1.99	1.94
87	3.95	3.10	2.71	2.48	2.32	2.21	2.12	2.05	1.99	1.94
88	3.95	3.10	2.71	2.48	2.32	2.20	2.12	2.05	1.99	1.94
89	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
91	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.98	1.94
92	3.95	3.10	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.94
93	3.94	3.09	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.93
94	3.94	3.09	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.93
95	3.94	3.09	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.93
96	3.94	3.09	2.70	2.47	2.31	2.20	2.11	2.04	1.98	1.93
97	3.94	3.09	2.70	2.47	2.31	2.19	2.11	2.04	1.98	1.93
98	3.94	3.09	2.70	2.47	2.31	2.19	2.10	2.03	1.98	1.93
99	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.98	1.93
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.98	1.93

Critical values of F for the 0.01 significance level:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	4052.19	4999.52	5403.34	5624.62	5763.65	5858.97	5928.33	5981.10	6022.50	6055.85
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.93	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.52	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.90	3.81
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.02	3.84	3.71	3.60	3.51
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.77	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.05	3.73	3.50	3.33	3.20	3.09	3.01
30	7.56	5.39	4.51	4.02	3.70	3.47	3.31	3.17	3.07	2.98
31	7.53	5.36	4.48	3.99	3.68	3.45	3.28	3.15	3.04	2.96
32	7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	3.02	2.93

33	7.47	5.31	4.44	3.95	3.63	3.41	3.24	3.11	3.00	2.91
34	7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98	2.89
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88
36	7.40	5.25	4.38	3.89	3.57	3.35	3.18	3.05	2.95	2.86
37	7.37	5.23	4.36	3.87	3.56	3.33	3.17	3.04	2.93	2.84
38	7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.92	2.83
39	7.33	5.19	4.33	3.84	3.53	3.31	3.14	3.01	2.90	2.81
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
41	7.30	5.16	4.30	3.82	3.50	3.28	3.11	2.98	2.88	2.79
42	7.28	5.15	4.29	3.80	3.49	3.27	3.10	2.97	2.86	2.78
43	7.26	5.14	4.27	3.79	3.48	3.25	3.09	2.96	2.85	2.76
44	7.25	5.12	4.26	3.78	3.47	3.24	3.08	2.95	2.84	2.75
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74
46	7.22	5.10	4.24	3.76	3.44	3.22	3.06	2.93	2.82	2.73
47	7.21	5.09	4.23	3.75	3.43	3.21	3.05	2.92	2.81	2.72
48	7.19	5.08	4.22	3.74	3.43	3.20	3.04	2.91	2.80	2.72
49	7.18	5.07	4.21	3.73	3.42	3.20	3.03	2.90	2.79	2.71
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.79	2.70
51	7.16	5.05	4.19	3.71	3.40	3.18	3.01	2.88	2.78	2.69
52	7.15	5.04	4.18	3.70	3.39	3.17	3.01	2.87	2.77	2.68
53	7.14	5.03	4.17	3.70	3.38	3.16	3.00	2.87	2.76	2.68
54	7.13	5.02	4.17	3.69	3.38	3.16	2.99	2.86	2.76	2.67
55	7.12	5.01	4.16	3.68	3.37	3.15	2.98	2.85	2.75	2.66
56	7.11	5.01	4.15	3.67	3.36	3.14	2.98	2.85	2.74	2.66
57	7.10	5.00	4.15	3.67	3.36	3.14	2.97	2.84	2.74	2.65
58	7.09	4.99	4.14	3.66	3.35	3.13	2.97	2.84	2.73	2.64
59	7.09	4.98	4.13	3.66	3.35	3.12	2.96	2.83	2.72	2.64
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
61	7.07	4.97	4.12	3.64	3.33	3.11	2.95	2.82	2.71	2.63
62	7.06	4.97	4.11	3.64	3.33	3.11	2.94	2.81	2.71	2.62
63	7.06	4.96	4.11	3.63	3.32	3.10	2.94	2.81	2.70	2.62
64	7.05	4.95	4.10	3.63	3.32	3.10	2.93	2.80	2.70	2.61
65	7.04	4.95	4.10	3.62	3.31	3.09	2.93	2.80	2.69	2.61
66	7.04	4.94	4.09	3.62	3.31	3.09	2.92	2.79	2.69	2.60
67	7.03	4.94	4.09	3.61	3.30	3.08	2.92	2.79	2.68	2.60
68	7.02	4.93	4.08	3.61	3.30	3.08	2.91	2.79	2.68	2.59
69	7.02	4.93	4.08	3.60	3.30	3.08	2.91	2.78	2.68	2.59
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59
71	7.01	4.92	4.07	3.60	3.29	3.07	2.90	2.77	2.67	2.58
72	7.00	4.91	4.07	3.59	3.28	3.06	2.90	2.77	2.66	2.58
73	7.00	4.91	4.06	3.59	3.28	3.06	2.90	2.77	2.66	2.57
74	6.99	4.90	4.06	3.58	3.28	3.06	2.89	2.76	2.66	2.57
75	6.99	4.90	4.05	3.58	3.27	3.05	2.89	2.76	2.65	2.57
76	6.98	4.90	4.05	3.58	3.27	3.05	2.88	2.76	2.65	2.56
77	6.98	4.89	4.05	3.57	3.27	3.05	2.88	2.75	2.65	2.56
78	6.97	4.89	4.04	3.57	3.26	3.04	2.88	2.75	2.64	2.56
79	6.97	4.88	4.04	3.57	3.26	3.04	2.87	2.75	2.64	2.55
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55
81	6.96	4.88	4.03	3.56	3.25	3.03	2.87	2.74	2.63	2.55
82	6.95	4.87	4.03	3.56	3.25	3.03	2.87	2.74	2.63	2.55

83	6.95	4.87	4.03	3.55	3.25	3.03	2.86	2.73	2.63	2.54
84	6.95	4.87	4.02	3.55	3.24	3.03	2.86	2.73	2.63	2.54
85	6.94	4.86	4.02	3.55	3.24	3.02	2.86	2.73	2.62	2.54
86	6.94	4.86	4.02	3.55	3.24	3.02	2.85	2.73	2.62	2.53
87	6.94	4.86	4.02	3.54	3.24	3.02	2.85	2.72	2.62	2.53
88	6.93	4.86	4.01	3.54	3.23	3.01	2.85	2.72	2.62	2.53
89	6.93	4.85	4.01	3.54	3.23	3.01	2.85	2.72	2.61	2.53
90	6.93	4.85	4.01	3.54	3.23	3.01	2.85	2.72	2.61	2.52
91	6.92	4.85	4.00	3.53	3.23	3.01	2.84	2.71	2.61	2.52
92	6.92	4.84	4.00	3.53	3.22	3.00	2.84	2.71	2.61	2.52
93	6.92	4.84	4.00	3.53	3.22	3.00	2.84	2.71	2.60	2.52
94	6.91	4.84	4.00	3.53	3.22	3.00	2.84	2.71	2.60	2.52
95	6.91	4.84	4.00	3.52	3.22	3.00	2.83	2.70	2.60	2.51
96	6.91	4.83	3.99	3.52	3.21	3.00	2.83	2.70	2.60	2.51
97	6.90	4.83	3.99	3.52	3.21	2.99	2.83	2.70	2.60	2.51
98	6.90	4.83	3.99	3.52	3.21	2.99	2.83	2.70	2.59	2.51
99	6.90	4.83	3.99	3.52	3.21	2.99	2.83	2.70	2.59	2.51
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50

Critical Values of Studentized Range Distribution(q) for Familywise ALPHA = .05.

Denominator DF	Number of Groups (a.k.a. Treatments) 3	4	5	6	7	8	9	10
1	26.976	32.819	37.081	40.407	43.118	45.397	47.356	49.070
2	8.331	9.798	10.881	11.734	12.434	13.027	13.538	13.987
3	5.910	6.825	7.502	8.037	8.478	8.852	9.177	9.462
4	5.040	5.757	6.287	6.706	7.053	7.347	7.602	7.826
5	4.602	5.218	5.673	6.033	6.330	6.582	6.801	6.995
6	4.339	4.896	5.305	5.629	5.895	6.122	6.319	6.493
7	4.165	4.681	5.060	5.359	5.606	5.815	5.997	6.158
8	4.041	4.529	4.886	5.167	5.399	5.596	5.767	5.918
9	3.948	4.415	4.755	5.024	5.244	5.432	5.595	5.738
10	3.877	4.327	4.654	4.912	5.124	5.304	5.460	5.598
11	3.820	4.256	4.574	4.823	5.028	5.202	5.353	5.486
12	3.773	4.199	4.508	4.748	4.947	5.116	5.262	5.395
13	3.734	4.151	4.453	4.690	4.884	5.049	5.192	5.318
14	3.701	4.111	4.407	4.639	4.829	4.990	5.130	5.253
15	3.673	4.076	4.367	4.595	4.782	4.940	5.077	5.198
16	3.649	4.046	4.333	4.557	4.741	4.896	5.031	5.150
17	3.628	4.020	4.303	4.524	4.705	4.858	4.991	5.108
18	3.609	3.997	4.276	4.494	4.673	4.824	4.955	5.071
19	3.593	3.977	4.253	4.468	4.645	4.794	4.924	5.037
20	3.578	3.958	4.232	4.445	4.620	4.768	4.895	5.008
21	3.565	3.942	4.213	4.424	4.597	4.743	4.870	4.981
22	3.553	3.927	4.196	4.405	4.577	4.722	4.847	4.957
23	3.542	3.914	4.180	4.388	4.558	4.702	4.826	4.935
24	3.532	3.901	4.166	4.373	4.541	4.684	4.807	4.915
25	3.523	3.890	4.153	4.358	4.526	4.667	4.789	4.897
26	3.514	3.880	4.141	4.345	4.511	4.652	4.773	4.880
27	3.506	3.870	4.130	4.333	4.498	4.638	4.758	4.864
28	3.499	3.861	4.120	4.322	4.486	4.625	4.745	4.850
29	3.493	3.853	4.111	4.311	4.475	4.613	4.732	4.837
30	3.487	3.845	4.102	4.301	4.464	4.601	4.720	4.824
31	3.481	3.838	4.094	4.292	4.454	4.591	4.709	4.813
32	3.475	3.832	4.086	4.284	4.445	4.581	4.698	4.802
33	3.470	3.825	4.079	4.276	4.436	4.572	4.689	4.791
34	3.465	3.820	4.072	4.268	4.428	4.563	4.680	4.782
35	3.461	3.814	4.066	4.261	4.421	4.555	4.671	4.773
36	3.457	3.809	4.060	4.255	4.414	4.547	4.663	4.764
37	3.453	3.804	4.054	4.249	4.407	4.540	4.655	4.756
38	3.449	3.799	4.049	4.243	4.400	4.533	4.648	4.749
39	3.445	3.795	4.044	4.237	4.394	4.527	4.641	4.741
40	3.442	3.791	4.039	4.232	4.388	4.521	4.634	4.735
41	3.439	3.787	4.035	4.227	4.383	4.515	4.628	4.728
42	3.436	3.783	4.030	4.222	4.378	4.509	4.622	4.722
43	3.433	3.779	4.026	4.217	4.373	4.504	4.617	4.716
44	3.430	3.776	4.022	4.213	4.368	4.499	4.611	4.710
45	3.428	3.773	4.018	4.209	4.364	4.494	4.606	4.705
46	3.425	3.770	4.015	4.205	4.359	4.489	4.601	4.700
47	3.423	3.767	4.011	4.201	4.355	4.485	4.597	4.695
48	3.420	3.764	4.008	4.197	4.351	4.481	4.592	4.690
49	3.418	3.761	4.005	4.194	4.347	4.477	4.588	4.686
50	3.416	3.758	4.002	4.190	4.344	4.473	4.584	4.681

Critical Values of Studentized Range Distribution(q) for Familywise ALPHA = .05.

Denominator	Number of Groups (a.k.a. Treatments)							
DF	3	4	5	6	7	8	9	10
51	3.414	3.756	3.999	4.187	4.340	4.469	4.580	4.677
52	3.412	3.753	3.996	4.184	4.337	4.465	4.576	4.673
53	3.410	3.751	3.994	4.181	4.334	4.462	4.572	4.669
54	3.408	3.749	3.991	4.178	4.331	4.459	4.569	4.666
55	3.406	3.747	3.989	4.176	4.328	4.455	4.566	4.662
56	3.405	3.745	3.986	4.173	4.325	4.452	4.562	4.659
57	3.403	3.743	3.984	4.170	4.322	4.449	4.559	4.656
58	3.402	3.741	3.982	4.168	4.319	4.447	4.556	4.652
59	3.400	3.739	3.979	4.165	4.317	4.444	4.553	4.649
60	3.399	3.737	3.977	4.163	4.314	4.441	4.550	4.646
61	3.397	3.735	3.975	4.161	4.312	4.439	4.548	4.643
62	3.396	3.734	3.973	4.159	4.309	4.436	4.545	4.641
63	3.395	3.732	3.972	4.157	4.307	4.434	4.542	4.638
64	3.393	3.730	3.970	4.155	4.305	4.431	4.540	4.635
65	3.392	3.729	3.968	4.153	4.303	4.429	4.538	4.633
66	3.391	3.727	3.966	4.151	4.301	4.427	4.535	4.630
67	3.390	3.726	3.965	4.149	4.299	4.425	4.533	4.628
68	3.389	3.725	3.963	4.147	4.297	4.423	4.531	4.626
69	3.387	3.723	3.962	4.146	4.295	4.421	4.529	4.624
70	3.386	3.722	3.960	4.144	4.293	4.419	4.527	4.621
71	3.385	3.721	3.959	4.142	4.291	4.417	4.525	4.619
72	3.384	3.719	3.957	4.141	4.290	4.415	4.523	4.617
73	3.383	3.718	3.956	4.139	4.288	4.413	4.521	4.615
74	3.382	3.717	3.954	4.138	4.286	4.411	4.519	4.613
75	3.382	3.716	3.953	4.136	4.285	4.410	4.517	4.611
76	3.381	3.715	3.952	4.135	4.283	4.408	4.515	4.610
77	3.380	3.714	3.951	4.133	4.282	4.406	4.514	4.608
78	3.379	3.713	3.949	4.132	4.280	4.405	4.512	4.606
79	3.378	3.712	3.948	4.131	4.279	4.403	4.511	4.604
80	3.377	3.711	3.947	4.129	4.278	4.402	4.509	4.603
81	3.377	3.710	3.946	4.128	4.276	4.400	4.507	4.601
82	3.376	3.709	3.945	4.127	4.275	4.399	4.506	4.600
83	3.375	3.708	3.944	4.126	4.274	4.398	4.504	4.598
84	3.374	3.707	3.943	4.125	4.272	4.396	4.503	4.597
85	3.374	3.706	3.942	4.123	4.271	4.395	4.502	4.595
86	3.373	3.705	3.941	4.122	4.270	4.394	4.500	4.594
87	3.372	3.704	3.940	4.121	4.269	4.392	4.499	4.592
88	3.372	3.704	3.939	4.120	4.268	4.391	4.498	4.591
89	3.371	3.703	3.938	4.119	4.266	4.390	4.496	4.590
90	3.370	3.702	3.937	4.118	4.265	4.389	4.495	4.588
91	3.370	3.701	3.936	4.117	4.264	4.388	4.494	4.587
92	3.369	3.700	3.935	4.116	4.263	4.387	4.493	4.586
93	3.368	3.700	3.934	4.115	4.262	4.386	4.492	4.585
94	3.368	3.699	3.934	4.114	4.261	4.384	4.491	4.583
95	3.367	3.698	3.933	4.114	4.260	4.383	4.489	4.582
96	3.367	3.698	3.932	4.113	4.259	4.382	4.488	4.581
97	3.366	3.697	3.931	4.112	4.258	4.381	4.487	4.580
98	3.366	3.696	3.930	4.111	4.257	4.380	4.486	4.579
99	3.365	3.696	3.930	4.110	4.257	4.379	4.485	4.578
100	3.365	3.695	3.929	4.109	4.256	4.379	4.484	4.577